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A Study on the Decision-Making of Product Production Process Based on Dynamic Programming Model

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Abstract

In view of the sales situation of enterprise products, especially for electronic products, the equipment and assembly of internal parts of products and the quality of parts themselves, the impact on the performance of finished products is obvious. This paper analyzes the finished product production of an enterprise producing electronic products, and makes the best decision on how to optimize the production process through quality control management. Firstly, using the approximation principle of binomial distribution and normal distribution, the minimum detection times of 98 and 59 are calculated at 95% and 90% confidence levels, respectively, to verify the premise that the nominal value of the merchant does not exceed 10%. Then, by constructing a dynamic programming model, the detection process is subdivided into the detection of parts and finished products and the disassembly decision of unqualified products, and the optimal decision-making schemes in six cases are obtained. Furthermore, the dynamic programming model is extended to 5 stages and 16 steps, and the optimal solution of each step is comprehensively considered to obtain the optimal decision of the whole process. Finally, the Bayesian updating method is introduced to dynamically adjust the defective rate according to the number of defective products detected in real time, and the previous decision-making scheme is updated accordingly to realize the dynamic optimization of decision-making.

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1 Introduction

In order to maintain the best-selling status of their products, enterprises not only need to innovate to meet the growing diversified needs of consumers, but also ensure the high quality and reliability of their products in today's highly competitive electronic product market. Product

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quality is directly related to the brand image, customer satisfaction and market share of the enterprise. Therefore, quality control management plays a vital role in the production process. For enterprises producing electronic products, the internal parts of the product, the assembly process and the quality of the parts themselves are the key factors affecting the performance of the finished product. Any omission in these links may lead to a decline in product quality, which in turn affects the market competitiveness of products, especially in the assembly process [1]. If the quality of the parts used is not up to standard or the assembly process is not fine, it will directly affect the overall performance and user experience of the product.

Predecessors have made significant research contributions in various types of product quality control and production optimization decisions. Eusébio Nunes et al. [2] use multi-criteria decision-making method (MCDM) and acceptance sampling technology to design the final product quality control plan, optimize the product quality control process, and reduce the factory rate of unqualified products. Based on the multi-criteria decision analysis and sampling inspection method, Aylin Arinç et al. [3] designed a new quality control process for finished products, which effectively improved the efficiency of quality control and reduced the risk of unqualified products flowing to the market. William J. Welch et al. [4] proposed a new strategy combining computer modeling and Taguchi off-line quality control method, which effectively reduced the number of trials in the design process of very large scale integrated circuits, and realized the efficient prediction and optimization of product performance. Yi-Ting Huang et al. [5] proposed a product quality fault detection scheme based on classification and regression tree, which can identify the relationship between manufacturing process parameters and unqualified products in real time and online, and made up for the shortcomings of traditional fault detection system. Aiming at the limitation of traditional rotation algorithm in digital engineering quality control system, Yingda Chen [6] proposed an improved scheme based on dynamic programming method. Experiments show that it has significant advantages in improving simulation accuracy and processing speed. Pravin P. Tambe et al. [7] combined maintenance, production scheduling and quality control to minimize system operating costs by optimizing decision variables. Rodrigo Polo-Mendoza et al. [8] explored a multi-objective optimization method based on genetic algorithm to achieve economic and environmental sustainability of pavement material design. Based on the uncertain multi-objective decision-making strategy framework of uncertainty theory, Chen-Fu Chien et al. [9] provided effective decision support for the capacity expansion and migration of semiconductor manufacturing industry. Wen Zhao et al. [10] discussed the coordination of dynamic joint pricing production / ordering decisions between manufacturers and distributors in a decentralized supply chain, and proposed a forward algorithm to optimize distributors' decisions. Onur Kaya [11] studied the optimal strategy of manufacturers using raw materials and market returns to produce original and remanufactured goods under random demand, and designed contracts to coordinate the decentralized system. Umi Marfuah et al. [12] developed an overall production planning (APP) model under uncertainty, using dynamic programming (DP) method, combined with artificial neural network (ANN) technology for demand forecasting and fuzzy logic (FL) technology to optimize production costs. Xiaoqin Niu et al. [13] determined the best strategy to reduce carbon emissions through a novel fuzzy decision model (combined with q-ROFSs, M-SWARA and ELECTRE methods) and compared it with IFS and PFS.

In this paper, a best-selling electronic product produced by an enterprise is taken as an example. Through the systematic analysis of the design of parts sampling detection scheme, production decision optimization, the application of dynamic programming model and the influence of sampling detection uncertainty on decision-making, a set of comprehensive quality management decision-making framework is provided for enterprises. In this paper, the binomial distribution sampling method, combined with systematic sampling and simple random sampling strategy, is used to ensure the effectiveness and economy of detection. Through systematic integrations of quality, cost and risk and other influencing factors, provide enterprise with a optimal decision-making scheme; in the multi-stage production process, the dynamic programming model is used to flexibly respond to the state changes in the production process to ensure the optimality of the decision. Considering the influence of sampling detection uncertainty on decision-making, Bayesian updating is used to improve the robustness of decision-making. In summary, the research results of this paper provide strong theoretical support and practical guidance for enterprise quality management. All data in this article are from https://www.mcm.edu.cn/.

2 Design sampling detection scheme

2.1 Background introduction

In order to ensure that the product is qualified, the enterprise needs to design a reasonable sampling detection plan [14], so that the number of enterprise detection is as small as possible. The parts purchased from suppliers should meet the requirement that the defective rate should not exceed 10% of the nominal value, and the scheme and specific results should be given under two different confidence of 95% and 90%.

2.2 Establish a binomial distribution model

The binomial distribution describes the probability distribution of the number of successful (or failed) trials in a fixed number of independent replicates. For the enterprise production detection scenario, each detection of whether the parts are qualified can be regarded as a test, and the defective rate is the probability of failure, and each detection is independent of each other. Through binomial distribution, the probability of a specific number of defective products can be calculated under a given number of sampling times and defective rate. This helps enterprises to formulate sampling inspection plans based on reliability requirements (such as 95% or 90%), so as to effectively judge whether the defective rate of the whole batch of spare parts exceeds the nominal value. Suppose n samples are randomly selected from the parts provided by the supplier, and the probability of each sample being defective is p, then the number of defective products C in the n samples obeys the binomial distribution [15][16]:

$$C \sim B(n, p), \tag{1}$$

Where C denotes the number of defective products, n is the number of samples, p is the defective rate of products (the supplier claims that $p = 0.1$).

2.3 Determine the sample size n under two levels of confidence [17] based on binomial distribution sampling methods [18]

Reject the spare parts at 95% confidence level:
Reject spare parts if the defective rate of spare parts exceeds 10% at 95% confidence level. Assume that the allowable error is E. When the sample size is large, the binomial distribution can be approximated using the normal distribution, so we use the normal distribution approximation to calculate the sample size n_1, at this point:

$$C \sim B(n, p) \to C \sim N(np, np(1-p)). \tag{2}$$

The sample size is estimated by using the sample size formula [19]. Sample size:

$$n = \frac{Z_\alpha^2 \cdot p \cdot (1-p)}{E^2}, \tag{3}$$

Of which:
Z_α is the critical value of the standard normal distribution at 95% confidence level – 1.645 (here is the one-side test);
p = 0.1 is the nominal value of defective rate;
E is the allowable error. In order to make the results more accurate, set a smaller E = 0.05.
Substituting the above parameters into the sample size formula, we can get:

$$n_1 = \frac{1.645^2 \cdot 0.1 \cdot (1 - 0.1)}{0.05^2} \approx 98, \tag{4}$$

Therefore, at least 98 samples need to be extracted to determine whether the defective rate exceeds 10% under the 95% confidence.
Receive spare parts at 90% confidence:
At 90% confidence level, if the defective rate of spare parts is less than or equal to 10%, the parts are received. Assuming that the allowable error is E, the normal approximation is used to calculate the sample size n_2, and the critical value Z_α = 1.28, other parameters are unchanged and brought into the formula:

$$n_2 = \frac{1.28^2 \cdot 0.1 \cdot (1 - 0.1)}{0.05^2} \approx 59, \tag{5}$$

Therefore, it is necessary to extract at least 59 samples to judge that the defective rate is not more than 10% under the confidence of 90%.

2.4 Sampling method combining systematic sampling and simple random sampling

Use simple random sampling design to determine whether to reject or accept the batch of parts according to the defective rate and the nominal value of the merchant under the condition of E = 0.05 and confidence of 95% and 90% respectively. The design is as follows: Given the total number of parts is s, set a classification ratio, denoted by k = 100 (k is the number of sample groups), roughly as follows:

Case 1: a total of s = 1000 parts, the number of samples in each group u_0 = s/k =10, then randomly select one in each group, a total of 100;
Case 2: a total of s = 10000 parts, the number of samples in each group u_0 = total number/k = 100, then randomly select one in each group, a total of 100;
Case 3: a total of s = 100,000 parts, the number of samples in each group u_0 = total number/k = 1000, then randomly select one in each group, a total of 100;
... (And so on)

In this way, 1 out of every 100 is always randomly selected. For example, if 98 are needed, then cycle through the first group again until 98 are taken.

2.5 Decision-making

For the 95% confidence condition: According to the method described in above, take 98 samples, record the number of defective parts and record it as c_1, then calculate the defective rate $k_{c1} = c_1/n_1$, if $k_{c1} > 10\%$, then reject the batch of parts;

For the 90% confidence condition: According to the method described in above, take 59 samples, record the number of defective parts and record it as c_2, then calculate the defective rate $k_{c2} = c_2/n_2$, if $k_{c2} \leq 10\%$, then accept the batch of parts.

Table 1: Enterprise production situation 1

Case	Part 1			Part 2		
	Defective rate	Purchase price	Detection cost	Defective rate	Purchase price	Detection cost
1	10%	4	2	10%	18	3
2	20%	4	2	20%	18	3
3	10%	4	2	10%	18	3
4	20%	4	1	20%	18	1
5	10%	4	8	20%	18	1
6	5%	4	2	5%	18	3

Case	Finished product			Unqualified product		
	Defective rate	Assembly cost	Detection cost	Market price	Exchange loss	Disassembly cost
1	10%	6	3	56	6	5
2	20%	6	3	56	6	5
3	10%	6	3	56	30	5
4	20%	6	2	56	30	5
5	10%	6	2	56	10	5
6	5%	6	3	56	10	40

3 Enterprise product production stage decision-making scheme

3.1 Background introduction

In the process of product production, enterprises need to make a series of decisions for each stage, combined with the relevant data of parts and finished products, such as whether to detect spare parts, whether to detect finished products, whether to disassemble unqualified products, etc. According to the determined decision-making scheme, the decision-making basis and evaluation index are given in detail. Enterprises need to purchase two kinds of parts (part 1, part 2), and produce finished products through the assembly process. The goal is to ensure product quality, reduce production costs and after-sales losses as much as possible, and design the optimal decision-making scheme [20]. The units of purchase price, detection cost, assembly cost, market price, exchange loss and disassembly cost are all RMB / piece.The details are as follows in Table 1:

3.2 Establish a dynamic programming model

Dynamic programming [21] is a method of solving complex problems by decomposing the original problem into relatively simple sub-problems. This problem requires a decision on whether to detect parts and finished products and whether to disassemble unqualified products in the production process of the enterprise. The decision is based on the defective rate and detection costs (disassembly costs). Based on the dynamic programming model, the decision-making process is divided into the following four steps: 1. Whether to detect part 1; 2. Whether to detect part 2; 3. Whether to detect the finished product; 4. Whether to disassemble the unqualified product (two cases: the unqualified products measured during the detection of finished products; the unqualified products exchanged back after sold); at each step the decision is made to detect or not (to disassemble or not), and the optimal solution (lowest cost) in the current state is recorded. Until the last step, where the optimal solution of full process is obtained. The specific process is shown in Figure 1:

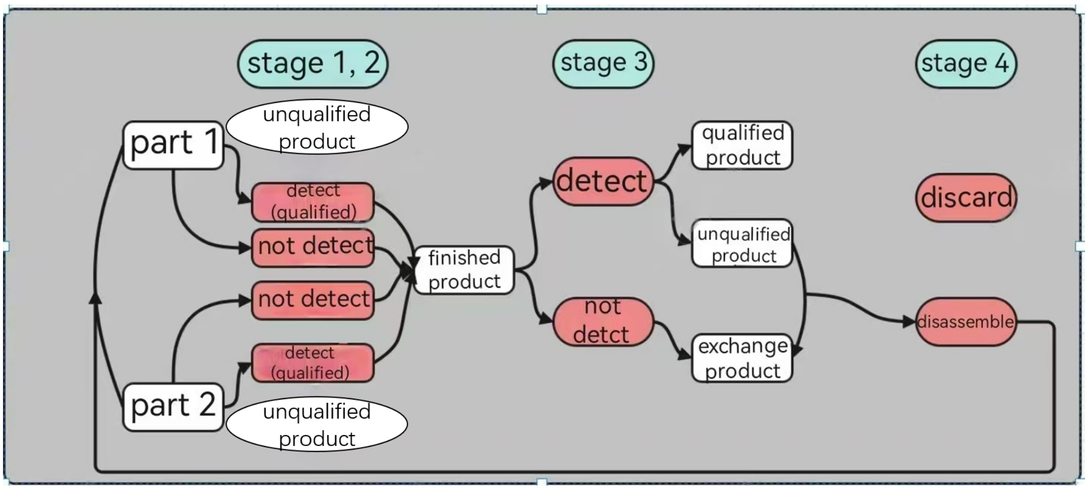

Figure 1: Flowchart

3.2.1 Establishment of dynamic programming model

1) Determine the factor indicators affecting the decision-making:

- Detection cost of part 1: C_{d1},
- Detection cost of part 2: C_{d2},
- Detection cost of finished product: C_{d3},
- Disassembly cost: C_{di},
- Exchange loss: C_e,
- Defective rate of spare part 1: p_1,
- Defective rate of part 2: p_2,
- Defective rate of finished product: p_3,

2) Definition of states:

- State 1: No detection and disassembly,
- State 2: Detection or disassembly,
- State j of the ith stage: $dp(i,j)$, such as the state 2 of the first stage is denoted as $dp(1,2)$, i.e., detection of part 1 ($i = 1,2,3,4$).

3) Specific modeling:

- Firstly, establish the overall cost equation:

$$dp(i,1) = \min(dp(i-1,1) + p_2 * C_e, dp(i-1,2) + p_2 * C_e), \tag{6}$$

$$dp(i,2) = \min(dp(i-1,1) + C_{d2}, dp(i-1,2) + C_{d2}), \tag{7}$$

  Equation (3.1) considers the cost of non-detection or non-disassembly. Equation (3.2) considers the cost of detection or disassembly.

- Stage 1:

$$dp(1,1) = \min(dp(0,1) + p_1 * C_e, dp(0,2) + p_1 * C_e), \tag{8}$$

$$dp(1,2) = \min(dp(0,1) + C_{d1}, dp(0,2) + C_{d1}), \tag{9}$$

  By default, $dp(0,j) = 0$. This stage calculates the cost of not detecting part 1 and the cost of detecting part 1.

- Stage 2:

$$dp(2,1) = \min(dp(1,1) + p_2 * C_e, dp(1,2) + p_2 * C_e), \tag{10}$$

$$dp(2,2) = \min(dp(1,1) + C_{d2}, dp(1,2) + C_{d2}), \tag{11}$$

This stage calculates the cost of not detecting part 2 and the cost of detecting part 2 based on the judged minimum cost of stage 1.

- Stage 3:

$$dp(3,1) = \min(dp(2,1) + p_3 * C_e, dp(2,2) + p_3 * C_e), \tag{12}$$

$$dp(3,2) = \min(dp(2,1) + C_{d3}, dp(2,2) + C_{d3}), \tag{13}$$

This stage calculates the cost of not detecting the finished product and the cost of detecting the finished product based on the judged minimum cost of stage 2.

- Stage 4:

$$dp(4,1) = \min(dp(3,1) + C_{di}, dp(3,2) + C_{di}), \tag{14}$$

$$dp(4,2) = \min(dp(3,1) + C_e, dp(3,2) + C_e), \tag{15}$$

$$dp = \min(dp(3,1) + C_e, dp(3,2) + C_e), \tag{16}$$

This stage calculates the cost of not disassembling the unqualified products as well as the cost of disassembling the unqualified products based on the judged minimum cost of stage 3, and ultimately selects the lower cost of the two states of this stage as the lowest total cost dp. At each step of the calculation, the selection is recorded, and backtracking is performed when the results are obtained.

3.3 Analyzing and solving each scenario using dynamic programming models

According to the indicators set, the original data were preprocessed into Table 2:

Table 2: Factor Indicators

Case	C_{d1}	C_{d2}	C_{df}	C_{di}	C_e	p_1	p_2	p_f
Case 1	2	3	3	5	6	10%	10%	10%
Case 2	2	3	3	5	6	20%	20%	20%
Case 3	2	3	3	5	30	10%	10%	10%
Case 4	1	1	2	5	30	20%	20%	20%
Case 5	8	1	2	5	10	10%	20%	10%
Case 6	2	3	3	40	10	5%	5%	5%

Bring the data from Table 2 into the model to calculate and obtain the decision results in Table 3:

Table 3: Results Of Decision-making For Situation 1

Case	Part 1: detecting	Part 2: detecting	Finished product: detecting	Finished product: disassembling
Case 1	No	No	No	No
Case 2	No	No	No	No
Case 3	Yes	No	No	No
Case 4	Yes	Yes	Yes	No
Case 5	No	Yes	No	No
Case 6	No	No	No	Yes

Analysis of results. According to the results of the model in Table 3, it can be shown:

- For Case 1, high revenue can be obtained at low cost by not detecting spare part 1, not detecting part 2, and not detecting and disassembling the finished product.
- For Case 3, in order to obtain the highest revenue at the lowest cost, detect part 1, do not detect part 2, do not detect the finished product, and do not disassemble the finished product. Other cases can be obtained from the table above.

4 The optimal decision of multi-stage production under specific conditions

4.1 Background introduction

Usually, the product needs to go through m processes, which are assembled from n parts, and the production decision-making scheme is required. In particular, if the enterprise needs to purchase 8 kinds of parts, assemble them into 3 kinds of semi-finished products, and assemble these 3 kinds of semi-finished products into finished products, the optimal decision-making scheme is designed. Enterprises need to find a balance between detection costs, assembly costs, disassembly costs and market losses to minimize total costs and improve product quality, select the best indicators, and achieve optimal decision-making [22]. This involves the problem of quality control in production management, especially how to reduce production and after-sales costs in the case of high defective rate. The details are as follows in Table 4:

Table 4: Enterprise Production Situation 2

Part	Defective rate	Purchase price	Detection cost
1	10%	2	1
2	10%	8	1
3	10%	12	2
4	10%	2	1
5	10%	8	1
6	10%	12	2
7	10%	8	1
8	10%	12	2

Semi-finished product	Defective rate	Assembly cost	Detection cost	Disassembly cost
1	10%	8	4	6
2	10%	8	4	6
3	10%	8	4	6
Finished product	10%	8	6	10

Note: Finished product, Market price: 200; Exchange loss: 40.

4.2 Establish a dynamic programming model

On the basis of the above, continue to use the dynamic programming model [23]. On the basis of the increase in the number of parts and processes, the model is expanded and the input parameters are added. Under the limitation of multiple parameters, better decision results can be obtained. From whether part 1 is detected to whether part 8 is detected, from whether semi-finished product 1 is detected to whether semi-finished product 3 is disassembled, and finally consider whether the finished product is detected and disassembled, a total of 16 decision-making processes. In order to clarify the specific process, it is divided into five stages: 1 to 8 steps, whether to detect parts; 9 to 11 steps, whether to detect semi-finished products; 12 to 14 steps, whether to disassemble unqualified semi-finished products; 15 step, whether to detect the finished products; 16 step, whether to disassemble the unqualified products. If the index of each stage is calculated separately, the steps will be too complicated and complicated. Therefore, the similar indicators are concentrated in the form of vectors to improve the efficiency of decision-making. Firstly, each indicator that affects the decision is introduced:

- Detection cost of 8 parts: C_{dp},
- Detection cost of 3 semi-finished products: C_{ds},
- Detection cost of finished products: C_{dpr},
- Disassembly cost of 3 semi-finished products: C_{dis},
- Disassembly cost of finished products: C_{dip},

- Exchange loss of defective products: C_e,
- Defective rate of parts: p_p,
- Defective rate of semi-finished products: p_s,
- Defective rate of finished products: p_{pr},

Record $dp(0) = 0$. For each stage i, calculate the cost of detection and the expected loss from non-detection (i = 1, 2, 3,. . . , 16). Record the optimal decision for each stage with the following equation:

$$dp(i+1) = dp(i) + min(C_{detected|disassembled}, C_{non-detection|non-disassembly}), \tag{17}$$

Stage 1: Steps 1 to 8, the decision is made whether or not to detect for parts, and the lowest cost is chosen by considering the cost of detecting parts and the expected value of the potential loss from not detecting:

$$dp(i+1) = dp(i) + min(C_{dp}, p_p * C_e), \tag{18}$$

Stage 2: Steps 9 to 11, make a decision on whether or not to detect the semi-finished product, choose the lowest cost by considering the cost of detecting the semi-finished product and the expected value of the possible loss from not testing:

$$dp(i+1) = dp(i) + min(C_{ds}, p_s * C_e), \tag{19}$$

Stage 3: Steps 12 to 14, decide whether to disassemble the semi-finished product or not, choose the lowest cost considering the cost of disassembling the semi-finished product and the expected value of the possible losses from non-disassembly:

$$dp(i+1) = dp(i) + min(C_{ds}, p_s * C_e), \tag{20}$$

Stage 4: Step 15, make a decision on whether or not to detect the finished product, choose the lowest cost by considering the cost of detecting the finished product and the expected value of the possible losses from not detecting:

$$dp(i+1) = dp(i) + min(C_{dpr}, p_p * C_e), \tag{21}$$

Stage 5: Step 16, decide whether or not to disassemble the finished product, choose the lowest cost by considering the cost of disassembling the finished product and the expected value of the possible loss from not disassembling it:

$$dp(i+1) = dp(i) + min(C_{dip}, p_p * C_e), \tag{22}$$

The $dp(17)$ obtained from the step 16 is the final minimum total cost. In each step of the calculation, record the choices made and backtrack after obtaining the results.

4.3 Analyzing and solving each scenario with dynamic programming models

Based on the indicators mentioned in above that influence the decision, retrieve the information from the questions and get:

- $C_{dp} = [2, 8, 12, 2, 8, 12, 8, 12]$,
- $C_{ds} = [8, 8, 8]$,
- $C_{dpr} = 8$,
- $C_{dis} = [6, 6, 6]$,
- $C_{dip} = 10$,
- $C_e = 40$,

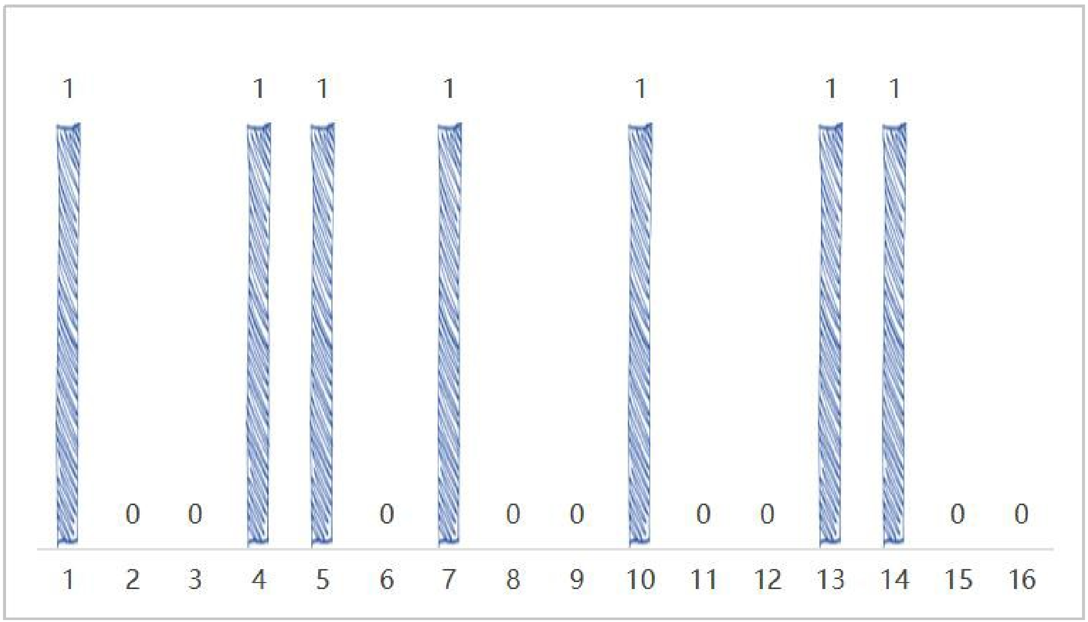

Figure 2: Summary Of Decision-making Outcomes

- $p_p = [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]$,
- $p_s = [0.1, 0.1, 0.1]$,
- $p_{pr} = 0.1$,

Bring the above data into the model for calculation, the final result is obtained as shown in Figure 2:

Note: Horizontal coordinates $1 \sim 8$ in the figure represent the decision of whether to detect part 1 to 8, $9 \sim 11$ represent the decision of whether to detect semi-finished product 1 to 3, $12 \sim 14$ represent the decision of whether to disassemble semi-finished products, 15 represent the decision of whether to detect finished products, and 16 represent the decision of whether to disassemble finished products, with the corresponding values 0 or 1. 1 means Yes. 0 means No. In order to visualize the connection between the decision-making results and the product production process, the following diagram is made in Figure 3:

Note: where 'green ✓' represents detection and 'red ×' represents non-detection; 'Green Thumbs Up' represents disassembly; 'Blue Thumbs Down' means non-disassembly.

5 The decision scheme is updated based on the premise of sampling detection

5.1 Background introduction

Under the premise of the previous sampling inspection scheme, after updating the defective rate, a new decision-making scheme is designed again for the table data in Table 1 and Table 4.

5.2 Bayesian core formula [24] (for calculating posterior probability)

$$P(\theta|X) = (P(X|\theta)P(\theta))/(P(X)), \tag{23}$$

Among:

- θ denotes the updated defective rate;
- P(θ) denotes the prior probability;
- P(θ |X) denotes the posterior probability, i.e., the probability of the parameter θ after the observed data X;

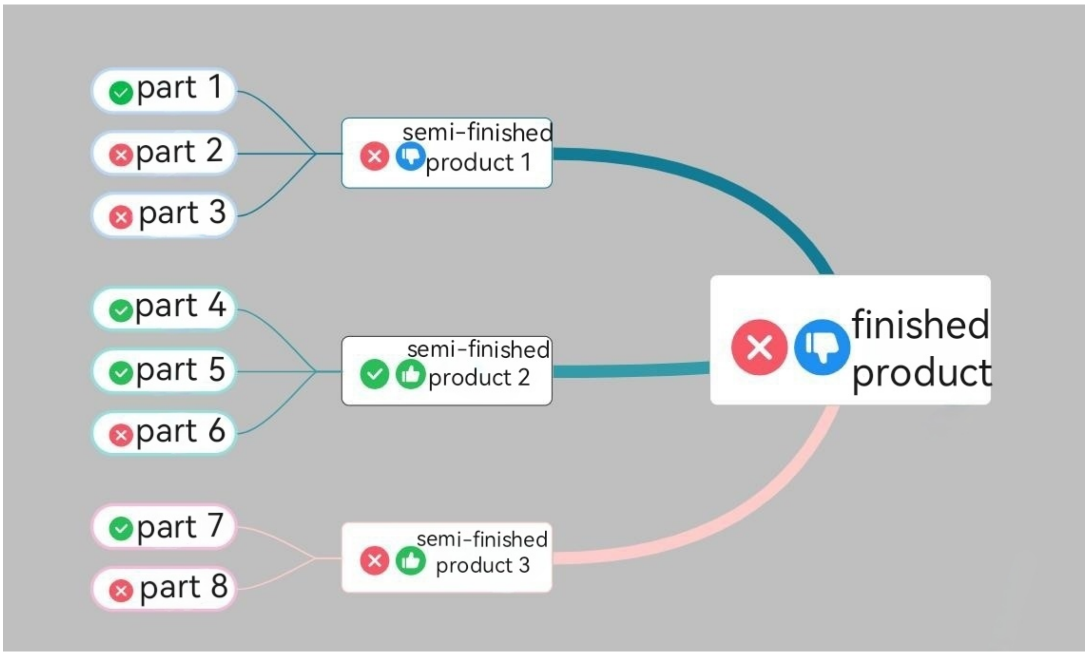

Figure 3: Situation 2 Results Chart

- P(X| θ) denotes the likelihood function, i.e., a function of the observed data X given the parameter θ.

5.3 Based on binomial distribution, use Bayesian formula [25, 26, 27, 28, 29, 30] for suboptimal rate estimation

1) Determine the prior distribution: choose the Beta distribution as the prior distribution (indicating the distribution of defective rate before sampling), and the defective rate of parts obeys the Beta distribution $\theta \sim Beta(\alpha, \beta)$.
2) Calculate the likelihood function:

$$P(k \mid \theta) = C_n^k \theta^k (1-\theta)^{n-k}, \tag{24}$$

where:

- n denotes the total number of samples taken;
- k denotes the number of sampled substandard products;
- θ denotes the current hypothetical substandard rate.

3) Determine the posterior distribution: $Beta(\alpha + k, \beta + n - k)$.
4) Obtain the new defective rate:

$$\theta_1 = \frac{\alpha + k}{\alpha + \beta + n}. \tag{25}$$

$$\theta_1 = (\alpha + k)/(\alpha + \beta + n). \tag{26}$$

5.4 Resolve situation 1

Taking the defective rate (10%) for part 1 in case 1 as an example, assume that with a sample size of 100, 10 defective products and 90 qualified products were drawn, taking 10% as the a priori probability. Assume that the sample size re-sampled is still 100, this time the extraction is of 13 defective products, 87 qualified products, then $k = 13$, the posterior distribution Beta$(23, 177)$,

$\theta_1 = 23/200$. Likewise, the defective rate of other parts or finished products after updating can be obtained, which is shown in Table 5:

Table 5: Comparison Of Old And New Defective Rate

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Original defective rate of part 1	0.1	0.2	0.1	0.2	0.1	0.05
New defective rate of part 1	0.115	0.215	0.115	0.215	0.115	0.065
Original defective rate of part 2	0.1	0.2	0.1	0.2	0.2	0.05
New defective rate of part 2	0.085	0.185	0.085	0.185	0.185	0.045
Original defective rate of finished product	0.1	0.2	0.1	0.2	0.1	0.05
New defective rate of finished product	0.11	0.21	0.11	0.21	0.11	0.06

Replacing the old defective rate with the new defective rate and repeating the process of situation 1, the new decision results are obtained as shown in Table 6:

Table 6: Outcome of Decision-making in Situation 1 (new)

Case	Part 1: detecting	Part 2: detecting	Finished product: detecting	Finished product: disassembling
Case 1	No	No	No	No
Case 2	No	No	No	No
Case 3	Yes	No	Yes	No
Case 4	Yes	Yes	Yes	No
Case 5	No	Yes	No	No
Case 6	No	No	No	Yes

5.5 Solving situation 2 again

The new defective rate is calculated in the same way as above and the following Table 7 is obtained:

Table 7: Comparison of Old And New Defective Rate

	Original defective rate	New defective rate		Original defective rate	New defective rate
Part 1	0.1	0.12	Part 7	0.1	0.115
Part 2	0.1	0.115	Part 8	0.1	0.105
Part 3	0.1	0.105	Part 1	0.1	0.115
Part 4	0.1	0.12	Part 2	0.1	0.115
Part 5	0.1	0.115	Part 3	0.1	0.115
Part 6	0.1	0.105	Finished product	0.1	0.095

Replacing the old defective rate with the new defective rate and repeating the process of situation 2, the new decision result is obtained as follows in Figure 4:

6 Discussion

Monte Carlo simulation is a method of estimating numerical results by repeated random sampling, which can be used to estimate the probability of an event or the expected value of a value. Therefore, it is selected to be applied to the test of the sampling method based on binomial distribution, and the performance of the model is tested by calculating the misjudgment rate of whether the parts are accepted or not.

Setting parameters, the nominal value of the defective rate of spare parts is 10%, the number of samples is 100, the number of tests, that is, the number of simulations, is 10000, and the counter is initialized to record the simulation results. For each simulation, the following operations are included: randomly selecting sample values from the binomial distribution; calculate the sample defect rate; update the counter or accumulator.

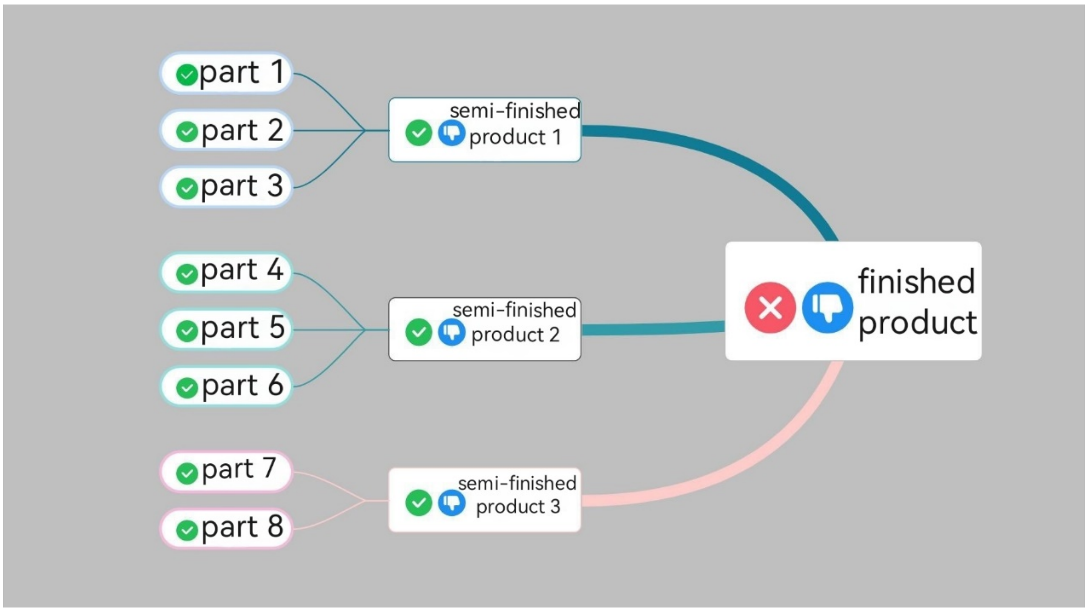

Figure 4: Results Of Situation 2 With New Defective Rate

In the repeated simulation, the defective rate is randomly generated according to the real defective rate range of $5\% - 20\%$, and the defective number sample is randomly generated based on the binomial distribution. The judgment is given by comparing the sample defective rate with the nominal value, and the validity of the sampling method is evaluated according to the simulation results. The acceptance misjudgment rate calculated by the above parameter settings is maintained at around 11%, and the rejection misjudgment rate fluctuates slightly around 5%. Therefore, it can be considered that the sampling method based on binomial distribution is more scientific.

Because this paper only makes decisions on the production process of a product in a specific enterprise, the research results have certain limitations. The use of simple binomial distribution may affect the accuracy to a certain extent. Further research needs to introduce more examples in the field to enhance the universality of the research results. In the actual enterprise quality control operation, it can be combined with genetic algorithm on the basis of dynamic programming, and the global search ability of genetic algorithm can be used to explore a wider range of solution space, improve model performance and decision-making efficiency.

7 Conclusion

In this paper, a dynamic programming decision model is established for the assembly detection of parts and unqualified products in the production process. Firstly, binomial sampling is used to determine the minimum number of detections under different confidence levels, and the sample size is calculated by using the normal distribution approximation. Then, a dynamic programming model is established, and the process is divided into steps such as whether to detect parts and finished products and whether to disassemble unqualified products. The optimal solution is recorded and the optimal decision-making scheme under different conditions is given. According to the change of defective rate, Bayesian is used to update the defective rate and re-decision, so as to realize dynamic programming.

8 Declarations

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Competing Interests

Not applicable.

Ethical Approval

Not applicable.

Authors's Contributions

Conceptualization, Y. Zhao; methodology, Y. Zhao and J. Huang; software, S. Wu; writing-original draft preparation, Y. Zhao; writing-review and editing, Y. Zhao and J. Leng.

Availability Data and Materials

Not applicable.

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