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On Computing Eccentricity Based Topological Invariants of Tickysim SpiNNaker Model Sheet

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Abstract

The study of topological invariants provides a dynamic way to establish a correlation with a structural graph and its attributes. In this form, nodes stand in for the vertices, and edges show the connections between them. In chemical graph theory substances are numerically modeled using topological indices to gain understanding of their physicochemical properties. Its eccentricity is the maximum distance of a random vertex, $\breve{a}$, and vertex, $\check{e}$, with minimum path. In this work, we compute the Tickysim SpiNNaker Model Sheet's eccentricity based indices. Further, we formulate analytically closed equations of these distance based topological invariants which support in examining the basic structural topology. The data analysis with graphs at certain points are developed using machine learning algorithms.

Keywords: Tickysim SpiNNaker Model Sheet, Eccentricity, GA_4, ABC_5
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1 Introduction

Graph theory is a branch of mathematics originating from a problem solved by Leonhard Euler in 1735, laying the foundation of modern graph theory. Chemical graph theory has roots in mathematical chemistry that employ the foundations of graph theory to model and explore molecular structure. In this approach, atoms are represented as vertices, and chemical bonds are represented as edges in a molecular graph. It helps in understanding molecular properties, predicting chemical behaviors, and designing new compounds. Various topological indices, such as the Wiener index and the Hosoya index, are used to quantify molecular characteristics and correlate them with physical, chemical, and biological properties. This theory is widely applied in cheminformatics, drug design, and nanotechnology, providing a mathematical foundation for exploring molecular interactions and optimizing chemical processes.

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TickySim and the SpiNNaker Model Sheet are powerful tools used in neuromorphic computing and large-scale neural network simulations. TickySim is a digital circuit simulator designed to model and analyze event-driven systems, particularly in neural networks and parallel computing architectures. On the other hand, SpiNNaker (Spiking Neural Network Architecture) Model Sheet provides a framework for simulating large-scale spiking neural networks, mimicking the human brain's function. These tools are widely used in fields such as neuroscience, for studying brain activity and disorders; artificial intelligence, for developing biologically inspired neural networks; robotics, for enhancing autonomous decision-making; and high-performance computing, for optimizing parallel processing architectures. Their ability to simulate complex neural interactions makes them essential for advancing brain-inspired computing technologies. TickySim SpiNNaker Model Sheet can be applied in Chemical Graph Theory (CGT) to model and analyze complex molecular structures using graph-based neural networks. SpiNNaker's spiking neural network framework can process these molecular graphs efficiently, simulating chemical reactions and predicting molecular properties based on topological indices. TickySim's event-driven simulation can be leveraged for reaction kinetics modeling, optimizing chemical pathways, and exploring molecular interactions in drug discovery. These tools enhance CGT applications by providing high-speed parallel computing capabilities, enabling real-time simulations of molecular behavior and assisting in the development of new materials and pharmaceutical.

Mathematical and chemical analysts are working in Cheminformatics at a rapid pace, which is widely used in both theoretical and computational chemistry. It uses graph theory to model chemical events mathematically. The study and experimentation of molecular structure is also conceivable through topological indices. Farooq et al. defined eccentricity indices of a hetrofunctional dendrimers [1] whereas Gao et al. focused on porphyrin-cored dendrimers [7]. Hayat et al. examined indices of honeycomb derived networks [11]. Iqbal et al. calculated indices of siloxane and POPAM dendrimers [13]. The authors focused on eccentricity based indices in [14, 29, 33, 34, 37]. Imran et al. computed indices of 2-power interconnection networks and symmetrical structure of bismuth tri-iodide [20, 21, 22, 23, 24]. Xu computed indices of some graphs in [36]. Sun et al. calculated quaternary synapses network for memristor-based spiking convolutional neural networks [31]. Further, in [15, 16, 17, 18, 19, 27, 28, 30, 35, 38, 40] the authors addressed some different types of indices. The length of the shortest path between two vertices $\check{a}$ and $\check{e}$, at a maximum distance, denoted by Œ, represents the eccentricity. The eccentricity of a given vertex $\check{a} \in V(P)$ is defined as $Œ(\check{a})=\max\{(\check{a},\check{e}); \forall\, \check{a} \in V(P)\}$.

The index of total eccentricity is defined as [22],

$$\xi(P) = \sum_{\check{a}\in V(P)} Œ(\check{a}). \tag{1}$$

Whereas, the eccentricity average is defined as [39],

$$avge(P) = \frac{1}{\eta}\sum_{\check{a}\in V(P)} Œ(\check{a}). \tag{2}$$

In [9], the authors have developed some new and modified versions of Zagreb indices of a molecular graph. The first, second and third Zagreb eccentricity indices are defined respectively as,

$$M_1^{*}(P) = \sum_{\check{a}\check{e}\in E(P)} [Œ(\check{a}) + Œ(\check{e})], \tag{3}$$

$$M_1^{**}(P) = \sum_{\check{a}\in V(P)} [Œ(\check{a})]^2, \tag{4}$$

$$M_2^*(P) = \sum_{\check{a}\check{e}\in E(P)} Œ(\check{a})Œ(\check{e}). \tag{5}$$

The geometric-arithmetic index based on eccentricity of a graph is proposed as [39],

$$GA_4(P) = \sum_{\check{a}\check{e}\in E(P)} \frac{2\sqrt{Œ(\check{a})Œ(\check{e})}}{Œ(\check{a})+Œ(\check{e})}. \tag{6}$$

Eccentricity based atom-bond connectivity index is established by Farahani [6], which is defined as,

$$ABC_5(P) = \sum_{\check{a}\check{e}\in E(P)} \sqrt{\frac{Œ(\check{a})+Œ(\check{e})-2}{Œ(\check{a})Œ(\check{e})}}. \tag{7}$$

2 Utilization of Topological Indices and Motivation

The first and second Zagreb indices have been found to represent the total π-electron energy of molecules [32, 34]. To accurately reflect key physicochemical properties, the GA index demonstrates relatively superior analytical capabilities compared to the predictive effectiveness of the Randic connectivity index [2, 8]. The ABC index provides an excellent correlation for assessing the strength of linear paraffins, branched alkanes, and for determining the strain energy of cycloalkanes [5, 29]. Degree-based topological indices play a crucial role in analyzing the chemical properties of various molecular structures. This study falls under the domain of quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) [3, 26, 28]. Motivated by these applications, we focus on computing eccentricity/ distance based topological indices, as they serve as fundamental tools for evaluating diverse structural properties [10, 25, 32, 36].

3 Method

To obtain our results, we employ techniques such as induction/generalization, the edge-splitting procedure, the vertex division method, a systematic approach, graph conceptual tools, and counting rules [4, 12]. Additionally, we utilize specific algorithms to visualize our mathematical findings.

4 Tickysim SpiNNaker Model Sheet

Our model sheet [19], comprises a total of $\ddot{m}\ddot{n}$ vertices and $3\ddot{m}\ddot{n} - 2\ddot{m} - 2\ddot{n} + 1$ edges. We focus on the scenario where $\ddot{m} = \ddot{n} = \check{\Lambda}$. In this case, the $\check{\Lambda} \times \check{\Lambda}$ structure, where $\check{\Lambda} \geq 3$, exhibits $\check{\Lambda}$ categories of vertices based on eccentricity and $2(\check{\Lambda} - 1)$ categories of edges classified by eccentricity. Table 1 presents the vertex partition and its corresponding count for the TSM sheet with $\check{\Lambda} \times \check{\Lambda}$ levels, categorized by eccentricity. Similarly, Table 2 displays the edge partition and its count for the TSM sheet with $\check{\Lambda} \times \check{\Lambda}$ levels, grouped by eccentricity. Figure 1, illustrates the TSM sheet with dimensions 12×12. Now, we focus on computing the closed formulas of indices for our network, P where $\check{\Lambda} \geq 3$.

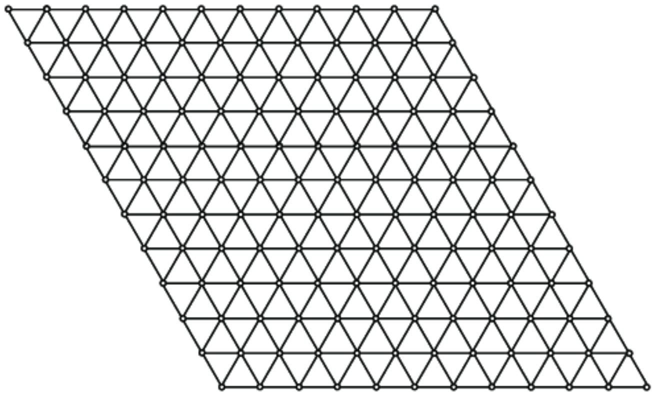


Figure 1: Tickysim SpiNNaker Model Sheet for $\curlywedge \times \curlywedge = 12 \times 12$

Theorem 4.1. *The index of total eccentricity is,*

$$\xi(P) \quad = \quad \curlywedge(\curlywedge - 1) + \sum_{\jmath,\ell}(2\curlywedge - \jmath)(\curlywedge + \ell)$$

Where; $\jmath = 2, 4, 6, 8, ...2(\curlywedge - 1)$ *and* $\ell = 0, 1, 2, 3, ...\curlywedge - 2$

Table 1: Types of Vertices and Their Count

$\text{Œ}(\check{a})$	$\lvert\text{Œ}(\check{a})\rvert$
$(\curlywedge - 1)$	$\curlywedge$
$(\curlywedge)$	$2\curlywedge - 2$
$(\curlywedge + 1)$	$2\curlywedge - 4$
$(\curlywedge + 2)$	$2\curlywedge - 6$
$(\curlywedge + 3)$	$2\curlywedge - 8$
.	.
.	.
.	.
$\curlywedge + (\curlywedge - 2)$	$2\curlywedge - 2(\curlywedge - 1)$

Table 2: Types of Edges and Their Count

$\text{Œ}(\check{a}), \text{Œ}(\check{e})$	$\lvert\text{Œ}(\check{a}), \text{Œ}(\check{e})\rvert$
$(\curlywedge - 1, \curlywedge - 1)$	$\curlywedge - 1$
$(\curlywedge - 1, \curlywedge)$	$4(\curlywedge - 1)$
$(\curlywedge, \curlywedge)$	$2(\curlywedge - 2)$
$(\curlywedge, \curlywedge + 1)$	$4(\curlywedge - 2)$
$(\curlywedge + 1, \curlywedge + 1)$	$2(\curlywedge - 3)$
$(\curlywedge + 1, \curlywedge + 2)$	$4(\curlywedge - 3)$
$(\curlywedge + 2, \curlywedge + 2)$	$2(\curlywedge - 4)$
$(\curlywedge + 2, \curlywedge + 3)$	$4(\curlywedge - 4)$
.	.
.	.
.	.
$(2\curlywedge - 3, 2\curlywedge - 3)$	$2(\curlywedge - (\curlywedge - 1))$
$(2\curlywedge - 3, 2\curlywedge - 2)$	$4(\curlywedge - (\curlywedge - 1))$

Proof. Utilizing the data from Table 1 in Equation 1, we derive the following result,

$$\begin{aligned}
\xi(P) &= \curlywedge(\curlywedge-1)+(2\curlywedge-2)(\curlywedge)+(2\curlywedge-4)(\curlywedge+1)+(2\curlywedge-6)(\curlywedge+2)+(2\curlywedge-8)(\curlywedge+3)\\
&\quad +...+(2\curlywedge-2(\curlywedge-1))(\curlywedge+(\curlywedge-2))\\
&= \curlywedge(\curlywedge-1)+\sum_{\jmath,\ell}(2\curlywedge-\jmath)(\curlywedge+\ell)
\end{aligned}$$

Where; $\jmath = 2,4,6,8,...2(\curlywedge-1)$ and $\ell = 0,1,2,3,...\curlywedge-2$

□

Theorem 4.2. *The eccentricity average is,*

$$avge(P) = 1-\frac{1}{\curlywedge}+\left[\frac{1}{\curlywedge^2}\sum_{\jmath,\ell}(2\curlywedge-\jmath)(\curlywedge+\ell)\right]$$

Where; $\jmath = 2,4,6,8,...2(\curlywedge-1)$ *and* $\ell = 0,1,2,3,...\curlywedge-2$

Proof. Utilizing the data from Table 1 in Equation 2, we derive the following result,

$$\begin{aligned}
avge(P) &= \frac{1}{\curlywedge^2}[\curlywedge(\curlywedge-1)]+\frac{1}{\curlywedge^2}[(2\curlywedge-2)(\curlywedge)+(2\curlywedge-4)(\curlywedge+1)+(2\curlywedge-6)(\curlywedge+2)\\
&\quad +(2\curlywedge-8)(\curlywedge+3)+(2\curlywedge-10)(\curlywedge+4)+...+(2\curlywedge-2(\curlywedge-1))(\curlywedge+(\curlywedge-2))]\\
&= 1-\frac{1}{\curlywedge}+\left[\frac{1}{\curlywedge^2}\sum_{\jmath,\ell}(2\curlywedge-\jmath)(\curlywedge+\ell)\right]
\end{aligned}$$

Where; $\jmath = 2,4,6,8,...2(\curlywedge-1)$ and $\ell = 0,1,2,3,...\curlywedge-2$

□

Theorem 4.3. *The first Zagreb eccentricity index is,*

$$M_1^*(P) = 2(\curlywedge-1)^2+4\sum_{\jmath,\ell}(\curlywedge-\jmath)(2\curlywedge+\ell)+2\sum_{p,q}(\curlywedge-p)(2\curlywedge+q)$$

Where; $\jmath = 1,2,3,4,...\curlywedge-1,\ \ \ell = -1,1,3,5,...,\ \ p = 2,3,4,5,...\curlywedge-1,$
and $q = 0,2,4,6,...$

Proof. Utilizing the data from Table 2 in Equation 3, we derive the following result,

$$\begin{aligned}
M_1^*(P) &= (\curlywedge-1)[\curlywedge-1+\curlywedge-1]+4(\curlywedge-1)[\curlywedge-1+\curlywedge]+2(\curlywedge-2)[\curlywedge+\curlywedge]\\
&\quad +4(\curlywedge-2)[\curlywedge+\curlywedge+1]+2(\curlywedge-3)[\curlywedge+1+\curlywedge+1]+4(\curlywedge-3)[\curlywedge+1+\curlywedge+2]\\
&\quad +...+4(\curlywedge-(\curlywedge-1))[2\curlywedge-3+2\curlywedge-2]\\
&= (\curlywedge-1)(2\curlywedge-2)+(4\curlywedge-4)(2\curlywedge-1)+(2\curlywedge-4)(2\curlywedge)+(4\curlywedge-8)(2\curlywedge+1)\\
&\quad +(2\curlywedge-6)(2\curlywedge+2)+(4\curlywedge-12)(2\curlywedge+3)+...+4(4\curlywedge-5)\\
&= 2(\curlywedge-1)^2+4\sum_{\jmath,\ell}(\curlywedge-\jmath)(2\curlywedge+\ell)+2\sum_{p,q}(\curlywedge-p)(2\curlywedge+q)
\end{aligned}$$

Where; $\jmath = 1,2,3,4,...\curlywedge-1,\ \ \ell = -1,1,3,5,...,\ \ p = 2,3,4,5,...\curlywedge-1,$
and $q = 0,2,4,6,...$

□

Theorem 4.4. *The second Zagreb eccentricity index is,*

$$M_1^{**}(P) = \Lambda(\Lambda-1)^2 + \sum_{\jmath,\ell}(2\Lambda-\jmath)[(\Lambda+\ell)]^2$$

Where; $\jmath = 2,4,6,8,...2(\Lambda-1)$ *and* $\ell = 0,1,2,3,...\Lambda-2$

Proof. Utilizing the data from Table 1 in Equation 4, we derive the following result,

$$\begin{aligned} M_1^{**}(P) &= \Lambda[\Lambda-1]^2 + (2\Lambda-2)[\Lambda]^2 + (2\Lambda-4)[\Lambda+1]^2 + (2\Lambda-6)[\Lambda+2]^2 \\ &\quad +(2\Lambda-8)[\Lambda+3]^2 + ... + (2\Lambda-2(\Lambda-1))[\Lambda+(\Lambda-2)]^2 \\ &= \Lambda(\Lambda-1)^2 + \sum_{\jmath,\ell}(2\Lambda-\jmath)[(\Lambda+\ell)]^2 \end{aligned}$$

Where; $\jmath = 2,4,6,8,...2(\Lambda-1)$ and $\ell = 0,1,2,3,...\Lambda-2$

□

Theorem 4.5. *The third Zagreb eccentricity index is,*

$$M_2^{*}(P) = (\Lambda-1)^3 + 4\sum_{\imath,\jmath,\ell}(\Lambda-\imath)(\Lambda+\jmath)(\Lambda+\ell) + 2\sum_{p,q}(\Lambda-p)(\Lambda+q)^2$$

Where; $\imath = 1,2,3,4,...\Lambda-1,\ \ \jmath = -1,0,1,2,3,...\Lambda-3,\ \ell = 0,1,2,3,...\Lambda-2,$
$p = 2,3,4,5,...\Lambda-1,$ *and* $q = 0,1,2,,3,...\Lambda-3$

Proof. Utilizing the data from Table 2 in Equation 5, we derive the following result,

$$\begin{aligned} M_2^{*}(P) &= (\Lambda-1)[(\Lambda-1)(\Lambda-1)] + 4(\Lambda-1)[(\Lambda-1)(\Lambda)] + 2(\Lambda-2)[(\Lambda)(\Lambda)] \\ &\quad +4(\Lambda-2)[(\Lambda)(\Lambda+1)] + 2(\Lambda-3)[(\Lambda+1)(\Lambda+1)] + 4(\Lambda-3)[(\Lambda+1)(\Lambda+2)] \\ &\quad +2(\Lambda-4)[(\Lambda+2)(\Lambda+2)] + 4(\Lambda-4)[(\Lambda+2)(\Lambda+3)] \\ &\quad +... + 4(\Lambda-(\Lambda-1))[(2\Lambda-3)(2\Lambda-2)] \\ &= (\Lambda-1)^3 + 4\sum_{\imath,\jmath,\ell}(\Lambda-\imath)(\Lambda+\jmath)(\Lambda+\ell) + 2\sum_{p,q}(\Lambda-p)(\Lambda+q)^2 \end{aligned}$$

Where; $\imath = 1,2,3,4,...\Lambda-1,\ \ \jmath = -1,0,1,2,3,...\Lambda-3,\ \ell = 0,1,2,3,...\Lambda-2,$
$p = 2,3,4,5,...\Lambda-1,$ and $q = 0,1,2,,3,...\Lambda-3$

□

Theorem 4.6. *The eccentricity based geometric arithmetic index is,*

$$GA_4(P) = (\Lambda-1) + 8\sum_{\imath,\jmath,\ell}\frac{(\Lambda-\imath)\sqrt{(\Lambda+\jmath)(\Lambda+\ell)}}{(\Lambda+\jmath)+(\Lambda+\ell)} + 2\sum_{p,q}\frac{(\Lambda-p)(\Lambda+q)}{(\Lambda+q)}$$

Where; $\imath = 1,2,3,4,...\Lambda-1,\ \ \jmath = -1,0,1,2,...\Lambda-3,\ \ \ell = 0,1,2,3,4,...\Lambda-2,$
$p = 2,3,4,5...\Lambda-1$ *and* $q = 0,1,2,3,...\Lambda-3$

Proof. Utilizing the data from Table 2 in Equation 6, we derive the following result,

$$
\begin{aligned}
GA_4(P) &= (\Lambda-1)\frac{2\sqrt{(\Lambda-1)(\Lambda-1)}}{\Lambda-1+\Lambda-1}+4(\Lambda-1)\frac{2\sqrt{(\Lambda-1)(\Lambda)}}{\Lambda-1+\Lambda}\\
&+2(\Lambda-2)\frac{2\sqrt{(\Lambda)(\Lambda)}}{\Lambda+\Lambda}+4(\Lambda-2)\frac{2\sqrt{(\Lambda)(\Lambda+1)}}{\Lambda+\Lambda+1}\\
&+2(\Lambda-3)\frac{2\sqrt{(\Lambda+1)(\Lambda+1)}}{\Lambda+1+\Lambda+1}+4(\Lambda-3)\frac{2\sqrt{(\Lambda+1)(\Lambda+2)}}{\Lambda+1+\Lambda+2}\\
&+2(\Lambda-4)\frac{2\sqrt{(\Lambda+2)(\Lambda+2))}}{\Lambda+2+\Lambda+2}+4(\Lambda-4)\frac{2\sqrt{(\Lambda+2)(\Lambda+3)}}{\Lambda+2+\Lambda+3}\\
&+...+4(\Lambda-(\Lambda-1))\frac{2\sqrt{(2\Lambda-3)(2\Lambda-2)}}{2\Lambda-3+2\Lambda-2}\\
&= (\Lambda-1)+8\sum_{\imath,\jmath,\ell}\frac{(\Lambda-\imath)\sqrt{(\Lambda+\jmath)(\Lambda+\ell)}}{(\Lambda+\jmath)+(\Lambda+\ell)}+2\sum_{p,q}\frac{(\Lambda-p)(\Lambda+q)}{(\Lambda+q)}
\end{aligned}
$$

Where; $\imath = 1,2,3,4,...\Lambda-1,\ \ \jmath = -1,0,1,2,...\Lambda-3,\ \ \ell = 0,1,2,3,4,...\Lambda-2,$
$p = 2,3,4,5...\Lambda-1$ and $q = 0,1,2,3,...\Lambda-3$

□

Theorem 4.7. *The eccentricity based atom-bond connectivity index is,*

$$
\begin{aligned}
ABC_5(P) &= \sqrt{2(\Lambda-2)}+4\sum_{\imath,\jmath,\ell}(\Lambda-\imath)\sqrt{\frac{[(\Lambda+\jmath)+(\Lambda+\ell)]-2}{[(\Lambda+\jmath)(\Lambda+\ell)]}}\\
&+2\sum_{p,q}(\Lambda-p)\sqrt{\frac{2(\Lambda+q)-2}{(\Lambda+q)^2}}
\end{aligned}
$$

Where; $\imath = 1,2,3,4,...\Lambda-1,\ \ \jmath = -1,0,1,2,...\Lambda-3,\ \ell = 0,1,2,3,4,...\Lambda-2,$
$p = 2,3,4,5...\Lambda-1$ *and* $q = 0,1,2,3,...\Lambda-3,$

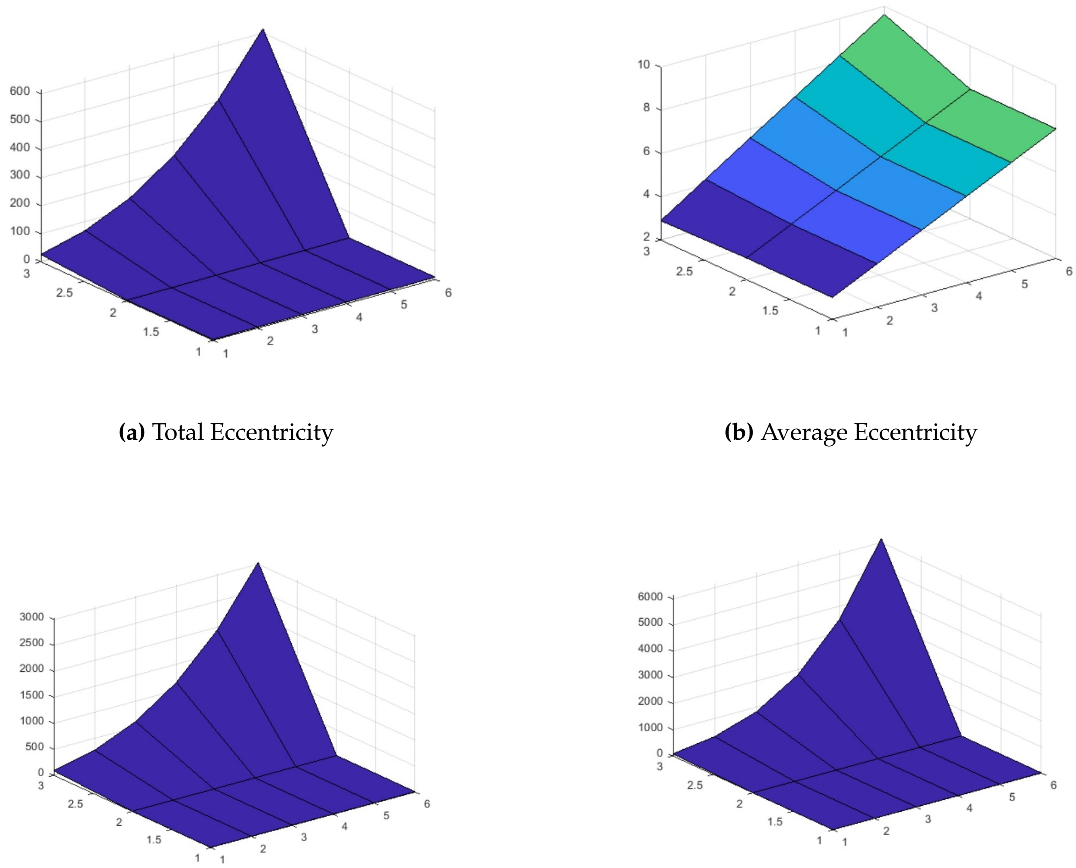
Proof. Utilizing the data from Table 2 in Equation 7, we derive the following result,

$$
\begin{aligned}
ABC_5(P) &= (\Lambda-1)\sqrt{\frac{(\Lambda-1+\Lambda-1)-2}{(\Lambda-1)(\Lambda-1)}}+4(\Lambda-1)\sqrt{\frac{(\Lambda-1+\Lambda)-2}{(\Lambda-1)(\Lambda)}}\\
&+2(\Lambda-2)\sqrt{\frac{(\Lambda+\Lambda)-2}{(\Lambda)(\Lambda)}}+4(\Lambda-2)\sqrt{\frac{(\Lambda+\Lambda+1)-2}{(\Lambda)(\Lambda+1)}}\\
&+2(\Lambda-3)\sqrt{\frac{(\Lambda+1+\Lambda+1)-2}{(\Lambda+1)(\Lambda+1)}}+4(\Lambda-3)\sqrt{\frac{(\Lambda+1+\Lambda+2)-2}{(\Lambda+1)(\Lambda+2)}}\\
&+2(\Lambda-4)\sqrt{\frac{(\Lambda+2+\Lambda+2)-2}{(\Lambda+2)(\Lambda+2)}}+4(\Lambda-4)\sqrt{\frac{(\Lambda+2+\Lambda+3)-2}{(\Lambda+2)(\Lambda+3)}}\\
&+...+4(\Lambda-(\Lambda-1))\sqrt{\frac{(2\Lambda-3+2\Lambda-2)-2}{(2\Lambda-3)(2\Lambda-2)}}\\
&= \sqrt{2(\Lambda-2)}+4\sum_{\imath,\jmath,\ell}(\Lambda-\imath)\sqrt{\frac{[(\Lambda+\jmath)+(\Lambda+\ell)]-2}{[(\Lambda+\jmath)(\Lambda+\ell)]}}\\
&+2\sum_{p,q}(\Lambda-p)\sqrt{\frac{2(\Lambda+q)-2}{(\Lambda+q)^2}}
\end{aligned}
$$

Where; $\imath = 1, 2, 3, 4, ...\curlywedge - 1$, $\jmath = -1, 0, 1, 2, ...\curlywedge - 3$, $\ell = 0, 1, 2, 3, 4, ...\curlywedge - 2$, $p = 2, 3, 4, 5...\curlywedge - 1$ and $q = 0, 1, 2, 3, ...\curlywedge - 3$,

□

5 Comparison and Discussion



(a) Total Eccentricity

(b) Average Eccentricity

(c) 1^{st} Zagreb Eccentricity

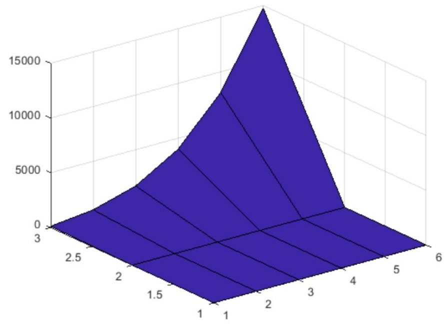
(d) 2^{nd} Zagreb Eccentricity

Figure 2: Comparison of Eccentricity-based Graph Indices

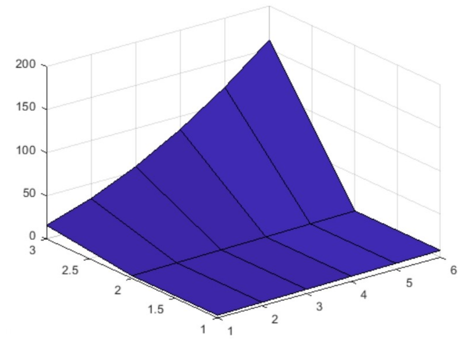
Utilizing machine learning algorithms we computed above indices with distinct values of $\curlywedge$ numerically for TSM sheet, which are presented in Table 3. The 3-D graphs of all these indices are shown in Figures 2a, 2b, 2c, 2d, 3a, 3b and 3c. From the data set we generated different models to compare and find the best one. The linear model is too simplistic and does not accurately describe the data set. The quadratic model is significantly better, showing a reasonable fit. The cubic model is the best approximation, closely matching the data set. If the goal is prediction accuracy, the cubic model should be preferred. However, for simplicity and interpretability, the quadratic model might be a good balance. Their 3-D graphs are also presented in Figures 4, 5, 6 and 7.

Table 3: TSM Sheet Indices for Certain Values of $[\breve{\lambda}, \breve{\lambda}]$

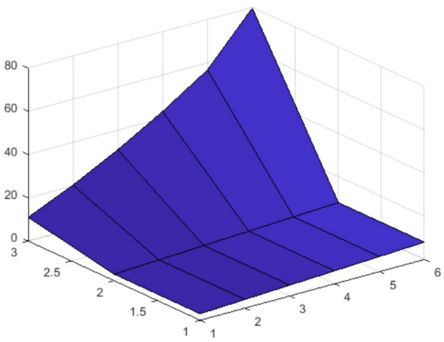
$[\breve{\lambda}, \breve{\lambda}]$	$\xi(P)$	$avec(P)$	$M_1^*(P)$	$M_1^{**}(P)$	$M_2^*(P)$	$GA_4(P)$	$ABC_5(P)$
[3, 3]	26	2.89	88	80	122	15.797	10.986
[4, 4]	68	4.25	270	304	565	32.811	20.251
[5, 5]	140	5.60	608	820	1696	55.819	31.001
[6, 6]	250	6.94	1150	1810	4005	84.824	43.088
[7, 7]	406	8.29	1944	3500	8114	119.827	56.688
[8, 8]	616	9.63	3038	6160	14777	160.829	79.688



(a) 3^{rd} Zagreb Eccentricity

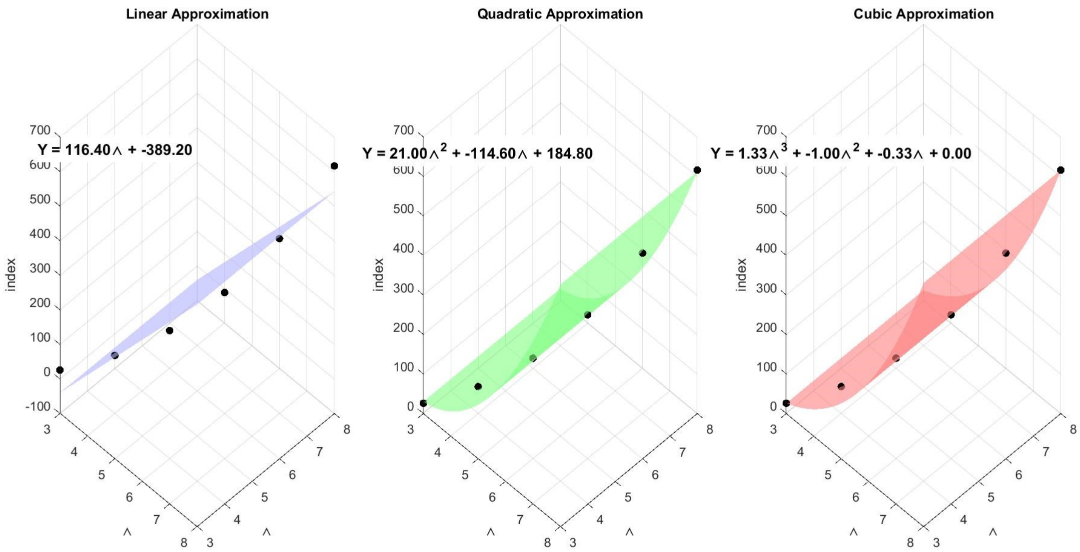


(b) GA_4

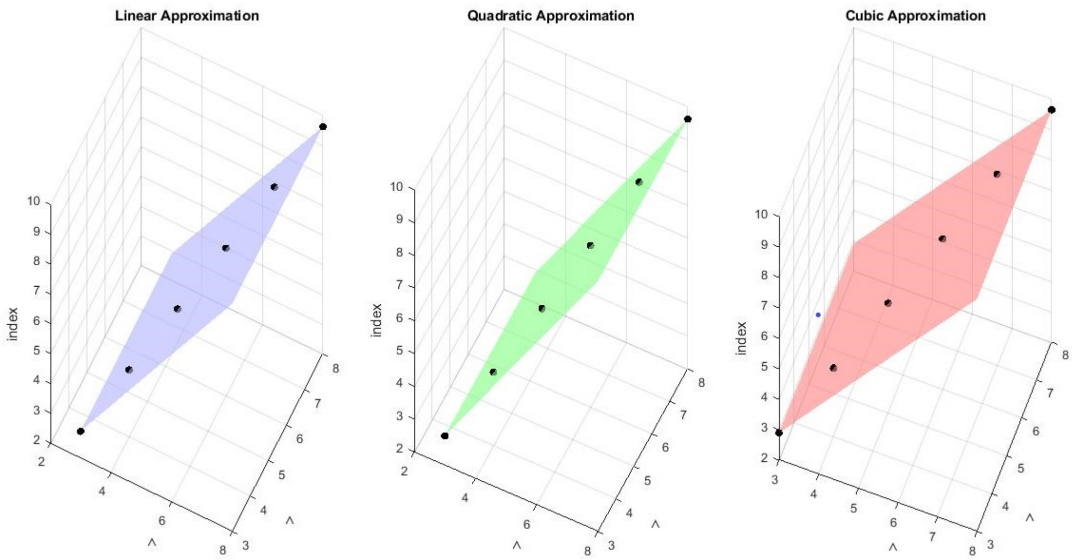


(c) ABC_5

Figure 3: Eccentricity-based and Degree-based Topological Indices

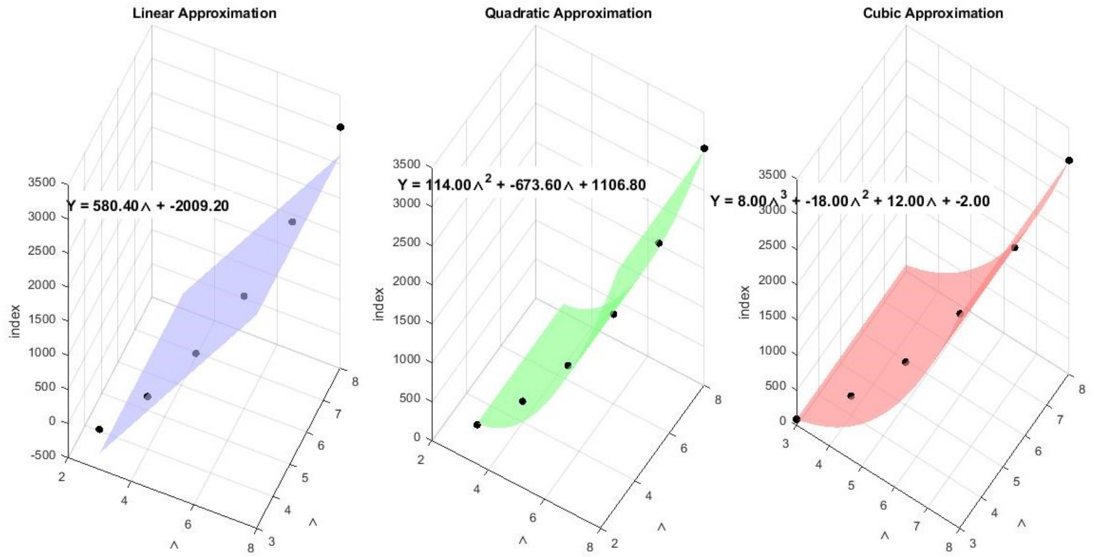


(a)

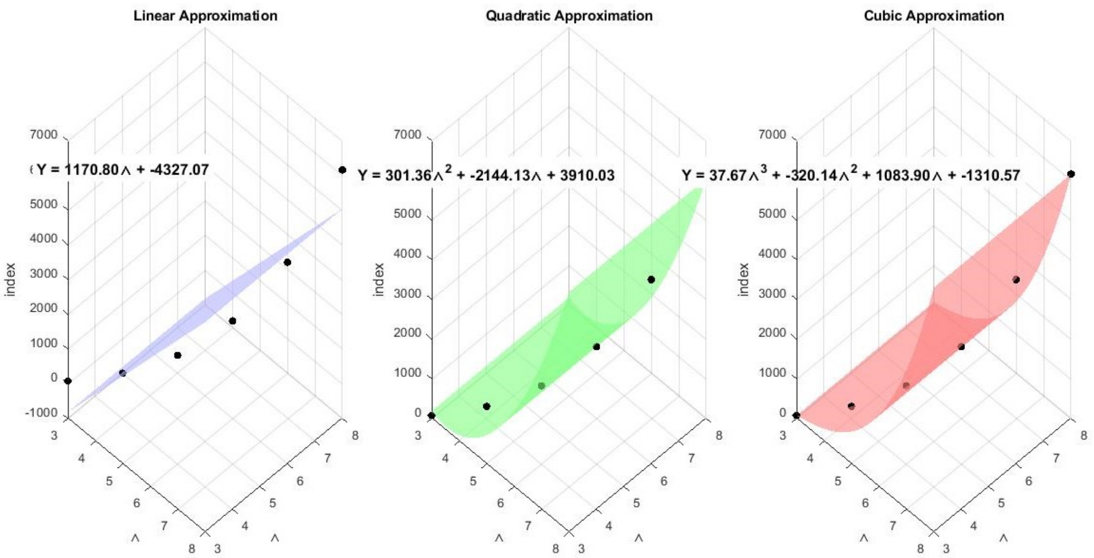


(b)

Figure 4: (a) Total Eccentricity Approximation, (b) Average Eccentricity Approximation

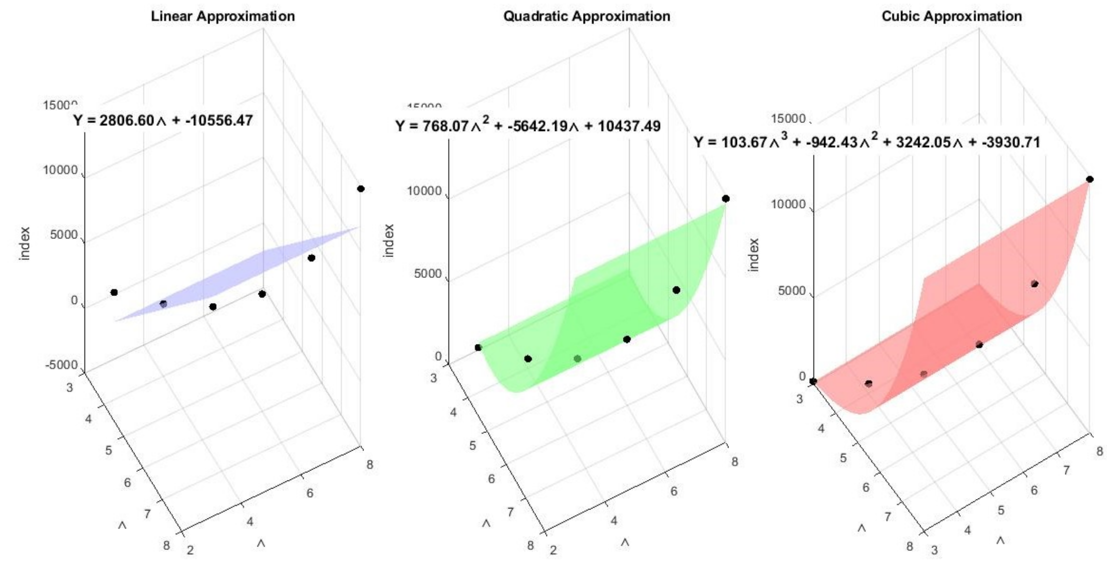


(a) 1^{st} Zagreb Eccentricity Approximation

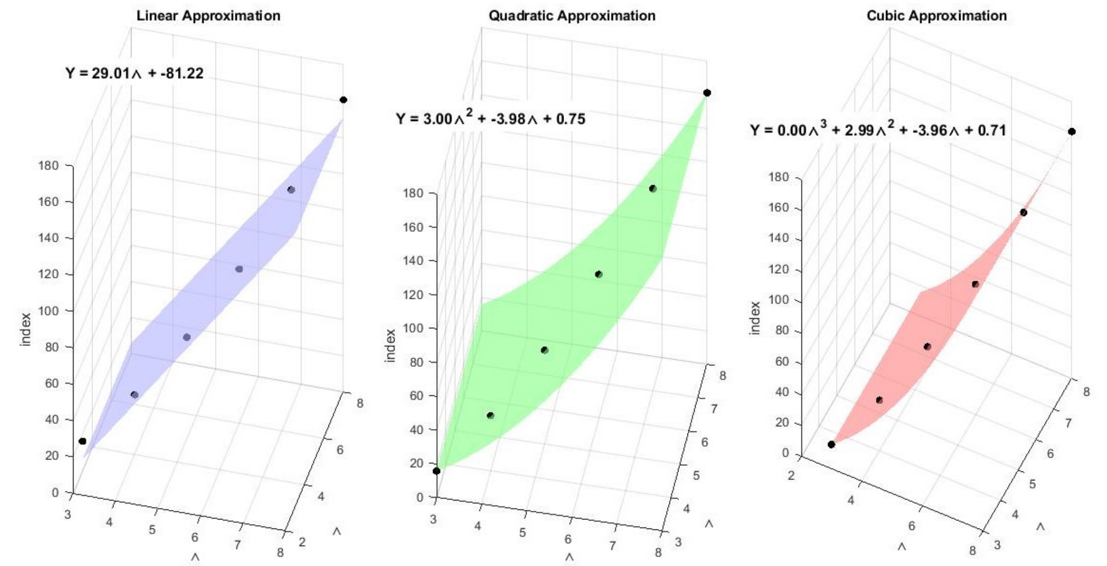


(b) 2^{nd} Zagreb Eccentricity Approximation

Figure 5: (a) 1^{st} Zagreb Eccentricity Approximation, (b) 2^{nd} Zagreb Eccentricity Approximation



(a)



(b)

Figure 6: (a) 3^{rd} Zagreb Eccentricity Approximation, (b) GA_4 Approximation

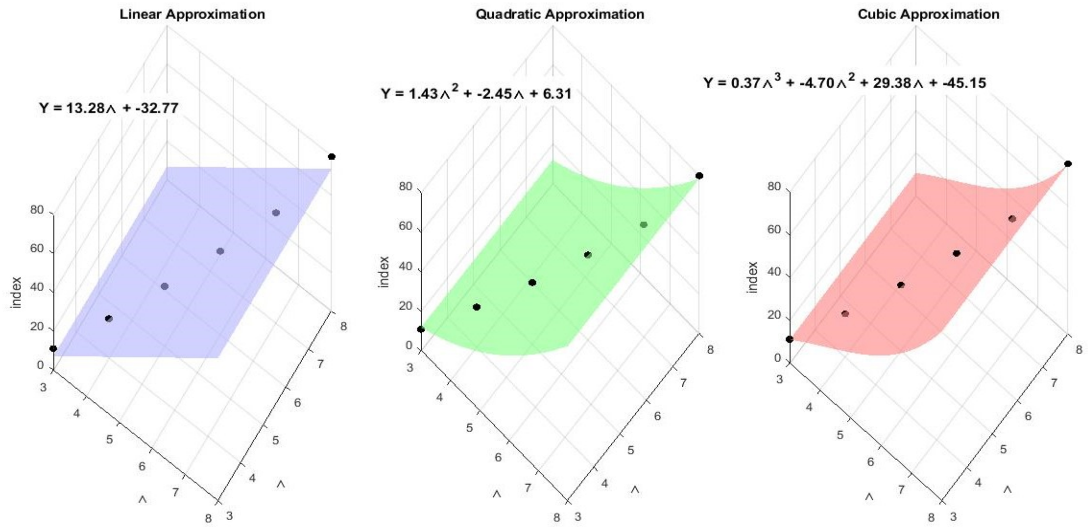


Figure 7: ABC_5 Approximation

6 Conclusions

In this article, we investigated various eccentricity-based topological indices for the Ticksysim SpiNNaker model sheet, including total eccentricity index, average eccentricity index, eccentricity-based Zagreb indices, eccentricity-based geometric-arithmetic index, and eccentricity-based atom bond connectivity index. We also analyzed their numerical behavior and visualized their trends graphically by using machine learning algorithms. Future research directions include exploring fundamental architectures and networks, and examining their distance-based topological invariants, which will provide valuable insights into the underlying topological structures.

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The authors declare that they have no competing interests.

Ethical Approval

Not applicable.

Authors's Contributions

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