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Non-uniform Trajectory Tracking AILC for Nonlinear Time-varying Parameter System with Multiple Unknown Control Directions

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Abstract

In this paper, a new iterative learning controller is presented for nonlinear time-varying parameter system to solve the tracking problem of different target trajectories. Based on Nussbaum function and the Lyapunov-like synthesis design the learning controller to handle system dynamics with multi-unknown control directions. Over a finite time interval, the unknown time-varying parameter is considered to be periodic, so it is expanded using a Fourier series expansion, and the remaining terms are treated with a canonical series. The controller designed in this paper can ensure that all signals of the closed-loop system are bounded in a finite time interval $[0, T]$, and can complete non-uniform target tracking. Finally, a simulation example is given to verify the effectiveness of the designed controller.

Key words: Iterative learning control, Time-varying parameter systems, Lyapunov-like, Multiple unknown control directions, Non-uniform target tracking
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1 Introduction

Adaptive Iterative Learning Control (AILC) is a control method for dealing with nonlinear repetitive trajectory tracking systems, it is an important branch of learning control. As the old saying goes, "If you repeat anything 10,000 times, you will become an expert", which is the basic idea of iterative learning control. Iterative learning control can obtain the control effect that conforms to the desired trajectory by continuously using the previous experimental information within a limited time interval. Due to its adaptability and easy to accomplish, it is used in many fields, such as drone [1], intelligent robot [2], multi-agent systems [3], high-speed trains [4], Motor [5] and so on.

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Nowadays, there are more and more researches on ILC, but most of the literatures stay on the control system with uniform tracking trajectory. For example, in [6], for the system with unknown constant parameter uncertainty, adaptive control and ILC Combined, an adaptive iterative learning control method based on Lyapunov technology is proposed to ensure the stability of the control system. In [7], a new control algorithm is proposed for high-order nonlinear systems, which can effectively deal with the target tracking problem of high-order systems. In [8], an iterative learning controller was designed for subway trains under speed and input constraints. The target trajectory is successfully tracked under the condition that the speed and input are never violated. In [9], an adaptive iterative learning control scheme based on neural network is established, which combines neural network with AILC to successfully solve the trajectory tracking problem of rigid robot manipulators with arbitrary initial errors. Aiming at the initial position problem of iterative learning control of the tank gun control system, the literature [10] proposed a construction method to correct the reference trajectory. The controller design adopts the quadratic obstacle Lyapunov function, so that the tank gun control system has excellent track performance. An non-uniform target tracking AILC method was proposed in the paper [11]. The paper [12] proposed a fault-tolerant ILC technique for mobile robots non-repetitive target tracking with output constraints. There are few relevant research results on the non-uniform objective control of the system.This is also a very important control issue.

In industry, most systems are more complex non-uniform control systems, and these complex systems often have many unknown control directions. There are few studies on control systems with multi-unknown control directions in the existing literature.In [13], An adaptive iterative learning control method of non-uniform trajectory tracking for strict feedback nonlinear time-varying systems with unknown control direction is proposed. In [14], a systematic procedure was developed to design a global adaptive control for a class of nonlinear systems, solving the problem of unknown control coefficients. This article leads to thinking, and the Nussbaum gain function is introduced into the ILC to solve the control system with unknown control direction in iterative learning control. Nussbaum gain function is a processing method proposed by Nussbaum when dealing with control systems with unknown control directions. For example, in [15], for the adaptive control method of time-varying uncertain nonlinear systems, a strict feedback form is used to solve the problem that the control system has unknown time-varying control coefficients. In [16], Nussbaum gain was introduced into iterative learning control for discrete-time systems with random initial conditions and trajectories required for iterative changes to solve the problem that discrete-time nonlinear systems have unknown control directions. However, so far, the different target trajectories tracking control problem of systems with multiple unknown control directions in a finite time interval has not been well solved.

Inspired by the above discussion, we propose a new nonlinear iterative learning controller handling non-uniform trajectory tracking problems for a class of nonlinear system with multi-unknown control direction. Use the Nussbaum gain function to handle unknown directions of the control system. A typical Fourier series with bounded residual terms is introduced into the control system to deal with time-varying parameters. The learning controller is designed based on the Lyapunov second method, which can effectively deal with the system dynamics with non-global Lipschitz nonlinearity, and can make the output result track the desired different trajectorys accurately. Finally, a simulation example is given and the simulation results are obtained. Combining theory and simulation, the feasibility and effectiveness of the experimental method are demonstrated.

2 System description

Given the following time-varying parameters systems

$$\begin{aligned}\dot{x}_{i,k} &= b_i x_{i+1,k} + \theta^T(t)\varphi_i(\bar{x}_{i,k}),\\ \dot{x}_{n,k} &= b_n u_k(t) + \theta^T(t)\varphi_n(\bar{x}_{n,k}),\\ y_k &= x_{1,k}.\end{aligned} \tag{2.1}$$

where $x_k = [x_{1,k}, \cdots, x_{n,k}]^T \in R^n$ is the measurable state vector, $\bar{x}_{i,k} = [x_{1,k}, \cdots, x_{i,k}]^T$, denoting $x_k = \bar{x}_{n,k}$, $u_k \in R$ is the system input, $y_k \in R$ is the system output. $\theta(t) \in R^p$ is an unknown time-varying function, the functions $\varphi_{i,k}(\bar{x}_{i,k})$, $i = 1, \cdots, n$ are known and smoothly, $\varphi_{i,k}(0) = 0$, $i = 1, \cdots, n$. All b_i, $i = 1, 2, \ldots, n$ are unknown, i.e., the control directions of the system are completely unknown.

Since a time-varying parameter can be regarded as periodic in a finite time interval,the processing method of unknown periodic time-varying parameters $\theta(t)$ is the same as in article [18]. By Fourier series, periodic function $\theta(t)$ can be expanded as

$$\theta(t) = \phi^T(t)\eta + \delta(t), |\delta(t)| \leq s, \tag{2.2}$$

where $\eta = [\eta_1, \eta_2, \cdots, \eta_q]^T \in R^q$ is a constant vector. And $\phi(t) = [\phi_1(t), \cdots, \phi_q(t)]^T$ with $\phi_1(t) = 1, \phi_{2j}(t) = \sin(\frac{2\pi jt}{T})$ and $\phi_{2j+1}(t) = \cos(\frac{2\pi jt}{T}), j = 1, \cdots, \frac{(q-1)}{2}$. s is the unknown bound. Substitute eq. (2.2) into the system (3.3), then

$$\begin{aligned}\dot{x}_{i,k} &= b_i x_{i+1,k} + \eta^T\phi(t)\varphi_i(\bar{x}_{i,k}) + \delta^T(t)\varphi_i(\bar{x}_{i,k}),\\ \dot{x}_{n,k} &= b_n u_k(t) + \eta^T\phi(t)\varphi_n(\bar{x}_{n,k}) + \delta^T(t)\varphi_n(\bar{x}_{n,k}),\\ y_k &= x_{1,k}.\end{aligned} \tag{2.3}$$

The control goal of this paper is to design an adaptive iterative learning control law $u_k(t)$ in a finite time interval $[0, T]$, which can complete the output of the system to track a given trajectory as the number k of iterations increases, i.e., when $k \to \infty$, $e_k(t) = y_k(t) - y_{r,k}(t) \to 0$. $y_{r,k}(t)$ is the given trajectory varies with iteration index k.

3 AILC design

This part uses the convergent series sequence [17] to deal with the upper bound of the remainder of the Fourier series expansion. The specific design process is as follows.

3.1 Design of the controller

Step 1. Denote $\omega_{1,k} = \varphi_{1,k}$, $S = s^2$, $B_1 = b_1^2$, and $|\delta^T(t)\varphi_{1,k}(x_{1,k})| \leq \|\delta^T(t)\|\bar{\varphi}_{1,k}(x_{1,k}) \leq s\bar{\varphi}_{1,k}(x_{1,k})$, where $\bar{\varphi}_{1,k}(x_{1,k}) > 0$; Take $z_{1,k} = x_{1,k} - y_{r,k}$, $z_{2,k} = x_{2,k} - \alpha_{1,k} - \dot{y}_{r,k}$, where $\alpha_{1,k}$ is the virtual controller. Derivativing $z_{1,k}$ by system (2.3) as following:

$$\dot{z}_{1,k} = b_1(z_{2,k} + \alpha_{1,k} + \dot{y}_{r,k}) + \eta^T\phi(t)\omega_{1,k} + \delta^T(t)\omega_{1,k} - \dot{y}_{r,k}. \tag{3.1}$$

For any real number and positive integer, Take $\Delta_k = \frac{a}{k^l}$, where $a > 0$, $l \geq 2$, $\tau_{1,k} = \Gamma_1\phi(t)\omega_{1,k}z_{1,k}$, $\nu_{1,k} = \Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{1,k}^2 z_{1,k}^2$. Take virtual control as $\alpha_{1,k} = N(\zeta_{1,k}(t))\psi_{1,k}(t) - \dot{y}_{r,k}$, $\psi_{1,k}(t) = c_1 z_{1,k} + \hat{\eta}_k^T\phi(t)\omega_{1,k} + \hat{S}_k\frac{1}{\Delta_k}\bar{\varphi}_{1,k}^2 z_{1,k} - \dot{y}_{r,k}$, $\dot{\zeta}_{1,k}(t) = \psi_{1,k}(t)z_{1,k}$, where $c_1 > \frac{1}{4}$ is a constant. Then eq. (3.1) becomes

$$\dot{z}_{1,k} = b_1 z_{2,k} + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) + \eta^T\phi(t)\omega_{1,k} + \delta^T(t)\omega_{1,k} - \dot{y}_{r,k}, \tag{3.2}$$

where $\hat{\eta}_k$, $\hat{S}_k$ are the estimated values of η, S, respectively. $\tilde{\eta}_k = \eta - \hat{\eta}_k$ and $\tilde{S}_k = S - \hat{S}_k$ are errors of the estimated parameters. Construct Lyapunov function as follows:

$$V_{1,k}(z_k, \hat{\eta}_k, \hat{S}_k) = \frac{1}{2}z_{1,k}^2 + \frac{1}{2}\tilde{\eta}_k^T\Gamma_1^{-1}\tilde{\eta}_k + \frac{1}{2}\Gamma_2^{-1}\tilde{S}_k^2, \tag{3.3}$$

where Γ_1 and Γ_2 are symmetric and positive definite matrices. Derivated $V_{1,k}$ by (3.2) as follows:

$$\begin{aligned}
\dot{V}_{1,k} = {} & b_1 z_{1,k} z_{2,k} + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) z_{1,k} + \eta^T\phi(t)\omega_{1,k}z_{1,k} + \\
& \delta^T(t)\omega_{1,k}z_{1,k} - \dot{y}_{r,k}z_{1,k} - \tilde{\eta}_k^T\Gamma_1^{-1}\dot{\hat{\eta}}_k - \Gamma_2^{-1}\tilde{S}_k\dot{\hat{S}}_k \\
& \leq b_1 z_{1,k} z_{2,k} + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) z_{1,k} + \hat{\eta}^T\phi(t)\omega_{1,k}z_{1,k} + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + s\bar{\varphi}_{1,k}|z_{1,k}| - \Gamma_2^{-1}\tilde{S}_k\dot{\hat{S}}_k - \dot{y}_{r,k}z_{1,k} \\
& \leq b_1 z_{1,k} z_{2,k} + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) z_{1,k} + \hat{\eta}^T\phi(t)\omega_{1,k}z_{1,k} + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + \frac{1}{\Delta_k}s^2\bar{\varphi}_{1,k}^2 z_{1,k}^2 + \frac{1}{4}\Delta_k - \Gamma_2^{-1}\tilde{S}_k\dot{\hat{S}}_k - \dot{y}_{r,k}z_{1,k} \\
& = b_1 z_{1,k} z_{2,k} + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) z_{1,k} + \hat{\eta}^T\phi(t)\omega_{1,k}z_{1,k} + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + \frac{1}{\Delta_k}S\bar{\varphi}_{1,k}^2 z_{1,k}^2 + \frac{1}{4}\Delta_k - \Gamma_2^{-1}\tilde{S}_k\dot{\hat{S}}_k - \dot{y}_{r,k}z_{1,k} \\
& = b_1 z_{1,k} z_{2,k} + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) z_{1,k} + \hat{\eta}^T\phi(t)\omega_{1,k}z_{1,k} + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + \frac{1}{\Delta_k}\hat{S}\bar{\varphi}_{1,k}^2 z_{1,k}^2 + \frac{1}{4}\Delta_k \\
& + \tilde{S}_k\Gamma_2^{-1}(\nu_{1,k} - \dot{\hat{S}}_k) - \dot{y}_{r,k}z_{1,k} \\
& = b_1 z_{1,k} z_{2,k} - c_1 z_{1,k}^2 + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) z_{1,k} + c_1 z_{1,k}^2 + \\
& \hat{\eta}^T\phi(t)\omega_{1,k}z_{1,k} + \frac{1}{\Delta_k}\hat{S}\bar{\varphi}_{1,k}^2 z_{1,k}^2 - \dot{y}_{r,k}z_{1,k} + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + \tilde{S}_k\Gamma_2^{-1}(\nu_{1,k} - \dot{\hat{S}}_k) + \frac{1}{4}\Delta_k \\
& = b_1 z_{1,k} z_{2,k} - c_1 z_{1,k}^2 + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) z_{1,k} + (c_1 z_{1,k} + \\
& \hat{\eta}^T\phi(t)\omega_{1,k} + \frac{1}{\Delta_k}\hat{S}\bar{\varphi}_{1,k}^2 z_{1,k} - \dot{y}_{r,k}) z_{1,k} + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + \tilde{S}_k\Gamma_2^{-1}(\nu_{1,k} - \dot{\hat{S}}_k) + \frac{1}{4}\Delta_k \\
& = b_1 z_{1,k} z_{2,k} - c_1 z_{1,k}^2 + b_1 N(\zeta_{1,k}(t))\psi_{1,k}(t) z_{1,k} + \psi_{1,k}(t) z_{1,k} + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + \tilde{S}_k\Gamma_2^{-1}(\nu_{1,k} - \dot{\hat{S}}_k) + \frac{1}{4}\Delta_k \\
& = b_1 z_{1,k} z_{2,k} - c_1 z_{1,k}^2 + (b_1 N(\zeta_{1,k}(t)) + 1)\psi_{1,k}(t) z_{1,k} + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + \tilde{S}_k\Gamma_2^{-1}(\nu_{1,k} - \dot{\hat{S}}_k) + \frac{1}{4}\Delta_k \\
& = b_1 z_{1,k} z_{2,k} - c_1 z_{1,k}^2 + (b_1 N(\zeta_{1,k}(t)) + 1)\dot{\zeta}_{1,k}(t) + \\
& \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k} - \dot{\hat{\eta}}_k) + \tilde{S}_k\Gamma_2^{-1}(\nu_{1,k} - \dot{\hat{S}}_k) + \frac{1}{4}\Delta_k \\
& = b_1^2 z_{2,k}^2 - (\frac{1}{2}z_{1,k} - b_1 z_{2,k})^2 - (c_1 - 1) z_{1,k}^2 \\
& - \frac{3}{4}z_{1,k}^2 + (b_1 N(\zeta_{1,k}(t)) + 1)\dot{\zeta}_{1,k}(t),
\end{aligned} \tag{3.4}$$

$$+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k}-\dot{\hat{\eta}}_k)+\tilde{S}_k\Gamma_2^{-1}(\nu_{1,k}-\dot{\hat{S}}_k)+\frac{1}{4}\Delta_k$$
$$\le B_1 z_{2,k}^2-(c_1-\frac{1}{4})z_{1,k}^2+(b_1N(\zeta_{1,k}(t))+1)\dot{\zeta}_{1,k}(t)+$$
$$\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k}-\dot{\hat{\eta}}_k)+\tilde{S}_k\Gamma_2^{-1}(\nu_{1,k}-\dot{\hat{S}}_k)+\frac{1}{4}\Delta_k. \tag{3.5}$$

The following inequality is exploited in the above derivation. $mn \le \frac{1}{r}m^2+\frac{1}{4}n^2r$, here $r>0$, $r=\Delta_k$. $B_1z_{2,k}^2$ will be canceled in the next step.

Step 2. Let $\omega_{2,k}=\varphi_{2,k}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}\varphi_{1,k}$, because $\varphi_{1,k},\varphi_{2,k}$ are known smooth functions, there exist smooth function $\bar{\varphi}_{2,k}(x_{1,k},x_{2,k},\hat{\eta}_k,\hat{S}_k)>0$ such that $|\delta^T(t)(\varphi_{2,k}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}\varphi_{1,k})|\le\|\delta^T(t)\|\bar{\varphi}_{2,k}\le s\bar{\varphi}_{2,k}$. Denote $B_2=b_2^2$; $z_{3,k}=x_{3,k}-\alpha_{2,k}-\ddot{y}_{r,k}$, $\tau_{2,k}=\tau_{1,k}+\Gamma_1\phi(t)\omega_{2,k}z_{2,k}$, $\nu_{2,k}=\nu_{1,k}+\Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{2,k}^2z_{2,k}^2$ then the time derivative of $z_{2,k}$ is given as following:

$$\dot{z}_{2,k}=b_2(z_{3,k}+\alpha_{2,k}+\ddot{y}_{r,k})+\eta^T\phi(t)\omega_{2,k}+\delta^T(t)\omega_{2,k}-\ddot{y}_{r,k}$$
$$-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}b_1x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k. \tag{3.6}$$

Construct Lyapunov function as follows:

$$V_{2,k}(z_k,\hat{\eta}_k,\hat{S}_k,\hat{B}_{1,k},\hat{b}_{1,k})=V_{1,k}+\frac{1}{2}z_{2,k}^2+\frac{1}{2}\Lambda_1^{-1}\tilde{B}_{1,k}^2+\frac{1}{2}\Upsilon_1^{-1}\tilde{b}_{1,k}^2, \tag{3.7}$$

where both Λ_1 and Υ_1 are symmetric and positive definite matrices. Derivated $V_{2,k}$ by (3.6) as follows:

$$\dot{V}_{2,k}=\dot{V}_{1,k}+z_{2,k}\dot{z}_{2,k}-\tilde{B}_{1,k}\Lambda_1^{-1}\dot{\hat{B}}_{1,k}-\tilde{b}_{1,k}\Upsilon_1^{-1}\dot{\hat{b}}_{1,k}$$
$$=\dot{V}_{1,k}+z_{2,k}(b_2(z_{3,k}+\alpha_{2,k}+\ddot{y}_{r,k})+\eta^T\phi(t)\omega_{2,k}+\delta^T(t)\omega_{2,k}$$
$$-\ddot{y}_{r,k}-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}b_1x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k)$$
$$-\tilde{B}_{1,k}\Lambda_1^{-1}\dot{\hat{B}}_{1,k}-\tilde{b}_{1,k}\Upsilon_1^{-1}\dot{\hat{b}}_{1,k}$$
$$\le B_1z_{2,k}^2-(c_1-\frac{1}{4})z_{1,k}^2+(b_1N(\zeta_{1,k}(t))+1)\dot{\zeta}_{1,k}(t)$$
$$+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k}-\dot{\hat{\eta}}_k)+\tilde{S}_k\Gamma_2^{-1}(\nu_{1,k}-\dot{\hat{S}}_k)+\frac{1}{4}\Delta_k+b_2z_{2,k}z_{3,k}$$
$$+b_2(\alpha_{2,k}+\ddot{y}_{r,k})z_{2,k}+z_{2,k}(\eta^T\phi(t)\omega_{2,k}+\delta^T(t)\omega_{2,k}$$
$$-\ddot{y}_{r,k}-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}b_1x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k)$$
$$-\tilde{B}_{1,k}\Lambda_1^{-1}\dot{\hat{B}}_{1,k}-\tilde{b}_{1,k}\Upsilon_1^{-1}\dot{\hat{b}}_{1,k}$$
$$\le-(c_1-\frac{1}{4})z_{1,k}^2+(b_1N(\zeta_{1,k}(t))+1)\dot{\zeta}_{1,k}(t)$$
$$+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{1,k}-\dot{\hat{\eta}}_k)+\tilde{S}_k\Gamma_2^{-1}(\nu_{1,k}-\dot{\hat{S}}_k)+\frac{1}{4}\Delta_k+b_2z_{2,k}z_{3,k}$$
$$+b_2(\alpha_{2,k}+\ddot{y}_{r,k})z_{2,k}+|\delta^T(t)\omega_{2,k}||z_{2,k}|+z_{2,k}(\eta^T\phi(t)\omega_{2,k}$$
$$-\ddot{y}_{r,k}-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}\hat{b}_{1,k}x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k+\hat{B}_{1,k}z_{2,k}),$$

$$
\begin{aligned}
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}-\dot{\hat{b}}_{1,k})\\
&\le-(c_1-\frac{1}{4})z_{1,k}^2+(b_1N(\zeta_{1,k}(t))+1)\dot{\zeta}_{1,k}(t)\\
&+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{2,k}-\dot{\hat{\eta}}_k)+\tilde{S}_k\Gamma_2^{-1}(\nu_{1,k}-\dot{\hat{S}}_k)+\frac{1}{4}\Delta_k+b_2z_{2,k}z_{3,k}\\
&+b_2(\alpha_{2,k}+\ddot{y}_{r,k})z_{2,k}+s\bar{\varphi}_{2,k}|z_{2,k}|+z_{2,k}(\hat{\eta}^T\phi(t)\omega_{2,k}\\
&-\ddot{y}_{r,k}-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}\hat{b}_{1,k}x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k+\hat{B}_{1,k}z_{2,k})\\
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}-\dot{\hat{b}}_{1,k})\\
&\le-(c_1-\frac{1}{4})z_{1,k}^2+(b_1N(\zeta_{1,k}(t))+1)\dot{\zeta}_{1,k}(t)\\
&+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{2,k}-\dot{\hat{\eta}}_k)+\tilde{S}_k\Gamma_2^{-1}(\nu_{1,k}-\dot{\hat{S}}_k)+\frac{1}{4}\Delta_k+b_2z_{2,k}z_{3,k}\\
&+b_2(\alpha_{2,k}+\ddot{y}_{r,k})z_{2,k}+\frac{1}{\Delta_k}s^2\bar{\varphi}_{2,k}^2z_{2,k}^2+\frac{1}{4}\Delta_k+z_{2,k}(\hat{\eta}^T\phi(t)\omega_{2,k}\\
&-\ddot{y}_{r,k}-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}\hat{b}_{1,k}x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k+\hat{B}_{1,k}z_{2,k})\\
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}-\dot{\hat{b}}_{1,k})\\
&=-(c_1-\frac{1}{4})z_{1,k}^2+(b_1N(\zeta_{1,k}(t))+1)\dot{\zeta}_{1,k}(t)+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{2,k}-\dot{\hat{\eta}}_k)\\
&+\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}(\tau_{2,k}-\dot{\hat{\eta}}_k)z_{2,k}+\tilde{S}_k\Gamma_2^{-1}(\nu_{1,k}-\dot{\hat{S}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}(\nu_{2,k}-\dot{\hat{S}}_k)z_{2,k}\\
&+\frac{2}{4}\Delta_k+b_2z_{2,k}z_{3,k}+b_2(\alpha_{2,k}+\ddot{y}_{r,k})z_{2,k}+\frac{1}{\Delta_k}S\bar{\varphi}_{2,k}^2z_{2,k}^2+z_{2,k}(\hat{\eta}^T\phi(t)\omega_{2,k}\\
&-\ddot{y}_{r,k}-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}\hat{b}_{1,k}x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\tau_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\nu_{2,k}+\hat{B}_{1,k}z_{2,k})\\
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}-\dot{\hat{b}}_{1,k})\\
&=-(c_1-\frac{1}{4})z_{1,k}^2+(b_1N(\zeta_{1,k}(t))+1)\dot{\zeta}_{1,k}(t)+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{2,k}-\dot{\hat{\eta}}_k)\\
&+\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}(\tau_{2,k}-\dot{\hat{\eta}}_k)z_{2,k}+\tilde{S}_k\Gamma_2^{-1}(\nu_{2,k}-\dot{\hat{S}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}(\nu_{2,k}-\dot{\hat{S}}_k)z_{2,k}\\
&+\frac{2}{4}\Delta_k+b_2z_{2,k}z_{3,k}+b_2(\alpha_{2,k}+\ddot{y}_{r,k})z_{2,k}+z_{2,k}(\hat{\eta}^T\phi(t)\omega_{2,k}+\frac{1}{\Delta_k}\hat{S}\bar{\varphi}_{2,k}^2z_{2,k}\\
&-\ddot{y}_{r,k}-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}\hat{b}_{1,k}x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\tau_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\nu_{2,k}+\hat{B}_{1,k}z_{2,k})\\
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}-\dot{\hat{b}}_{1,k}).
\end{aligned}\tag{3.8}
$$

Take virtual control as $\alpha_{2,k}=N(\zeta_{2,k}(t))\psi_{2,k}(t)-\ddot{y}_{r,k}$, $\psi_{2,k}(t)=c_2z_{2,k}+\hat{B}_{1,k}z_{2,k}+\hat{\eta}_k^T\phi(t)\omega_{2,k}+\hat{S}_k\frac{1}{\Delta_k}\bar{\varphi}_{2,k}^2z_{2,k}-\ddot{y}_{r,k}-\frac{\partial\alpha_{1,k}}{\partial t}-\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}\hat{b}_{1,k}x_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\tau_{2,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\nu_{2,k}$, $\dot{\zeta}_{2,k}(t)=\psi_{2,k}(t)z_{2,k}$, where $c_2>\frac{1}{4}$ is a constant, where $\hat{B}_{1,k}$, $\hat{b}_{1,k}$ are the estimation of parameters B_1, b_1, respectively. Both $\tilde{B}_{1,k}=B_1-\hat{B}_{1,k}$ and $\tilde{b}_{1,k}=b_1-\hat{b}_{1,k}$ are parameter estimation errors.

Then equation (3.8) can be rewrite as following:

$$
\begin{aligned}
\dot{V}_{2,k} \leq & -(c_1 - \frac{1}{4})z_{1,k}^2 + (b_1 N(\zeta_{1,k}(t)) + 1)\dot{\zeta}_{1,k}(t) + \tilde{\eta}_k^T \Gamma_1^{-1}(\tau_{2,k} - \dot{\hat{\eta}}_k) \\
& + \frac{\partial \alpha_{1,k}}{\partial \hat{\eta}_k}(\tau_{2,k} - \dot{\hat{\eta}}_k) z_{2,k} + \tilde{S}_k \Gamma_2^{-1}(\nu_{2,k} - \dot{\hat{S}}_k) + \frac{\partial \alpha_{1,k}}{\partial \hat{S}_k}(\nu_{2,k} - \dot{\hat{S}}_k) z_{2,k} \\
& + \frac{2}{4}\Delta_k + b_2 z_{2,k} z_{3,k} - c_2 z_{2,k}^2 + (b_2 N(\zeta_{2,k}(t)) + 1)\dot{\zeta}_{2,k}(t) \\
& + \tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2 - \dot{\hat{B}}_{1,k}) + \tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1 \frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{2,k} z_{2,k} - \dot{\hat{b}}_{1,k}) \\
= & -(c_1 - \frac{1}{4})z_{1,k}^2 + \sum_{i=1}^{2}(b_i N(\zeta_{i,k}(t)) + 1)\dot{\zeta}_{i,k}(t) + \tilde{\eta}_k^T \Gamma_1^{-1}(\tau_{2,k} - \dot{\hat{\eta}}_k) \\
& + \frac{\partial \alpha_{1,k}}{\partial \hat{\eta}_k}(\tau_{2,k} - \dot{\hat{\eta}}_k) z_{2,k} + \tilde{S}_k \Gamma_2^{-1}(\nu_{2,k} - \dot{\hat{S}}_k) + \frac{\partial \alpha_{1,k}}{\partial \hat{S}_k}(\nu_{2,k} - \dot{\hat{S}}_k) z_{2,k} \\
& + \frac{2}{4}\Delta_k + b_2^2 z_{3,k}^2 - (\frac{1}{2}z_{2,k} - b_2 z_{3,k})^2 - (c_2 - 1)z_{2,k}^2 - \frac{3}{4}z_{2,k}^2 \\
& + \tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2 - \dot{\hat{B}}_{1,k}) + \tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1 \frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{2,k} z_{2,k} - \dot{\hat{b}}_{1,k}) \\
\leq & -\sum_{i=1}^{2}(c_i - \frac{1}{4})z_{i,k}^2 + \sum_{i=1}^{2}(b_i N(\zeta_{i,k}(t)) + 1)\dot{\zeta}_{i,k}(t) + \tilde{\eta}_k^T \Gamma_1^{-1}(\tau_{2,k} - \dot{\hat{\eta}}_k) \\
& + \frac{\partial \alpha_{1,k}}{\partial \hat{\eta}_k}(\tau_{2,k} - \dot{\hat{\eta}}_k) z_{2,k} + \tilde{S}_k \Gamma_2^{-1}(\nu_{2,k} - \dot{\hat{S}}_k) + \frac{\partial \alpha_{1,k}}{\partial \hat{S}_k}(\nu_{2,k} - \dot{\hat{S}}_k) z_{2,k} \\
& + \tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2 - \dot{\hat{B}}_{1,k}) + \tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1 \frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{2,k} z_{2,k} - \dot{\hat{b}}_{1,k}) + \frac{2}{4}\Delta_k + B_2 z_{3,k}^2.
\end{aligned} \tag{3.9}
$$

Step 3. Let $\omega_{3,k} = \varphi_{3,k} - \sum_{i=1}^{2} \frac{\partial \alpha_{2,k}}{\partial x_{i,k}} \varphi_{i,k}$, because $\varphi_{i,k}, i = 1,2,3$ are known smooth functions, there exist smooth function $\bar{\varphi}_{3,k} > 0$ such that $|\delta^T(t)(\varphi_{3,k} - \sum_{i=1}^{2} \frac{\partial \alpha_{2,k}}{\partial x_{i,k}} \varphi_{i,k})| \leq \|\delta^T(t)\| \bar{\varphi}_{3,k} \leq s\bar{\varphi}_{3,k}$. Denote $B_3 = b_3^2$; $z_{4,k} = x_{4,k} - \alpha_{3,k} - y_{r,k}^{(3)}$, $\tau_{3,k} = \tau_{2,k} + \Gamma_1 \phi(t) \omega_{3,k} z_{3,k}$, $\nu_{3,k} = \nu_{2,k} + \Gamma_2 \frac{1}{\Delta_k} \bar{\varphi}_{3,k}^2 z_{3,k}^2$ then the time derivative of $z_{3,k}$ is given as following:

$$
\begin{aligned}
\dot{z}_{3,k} = & b_3(z_{4,k} + \alpha_{3,k} + y_{r,k}^{(3)}) + \eta^T \phi(t)\omega_{3,k} + \delta^T(t)\omega_{3,k} \\
& - y_{r,k}^{(3)} - \frac{\partial \alpha_{2,k}}{\partial t} - \sum_{i=1}^{2} \frac{\partial \alpha_{2,k}}{\partial x_{i,k}} b_i x_{i+1,k} \\
& - \frac{\partial \alpha_{2,k}}{\partial \hat{\eta}_k}\dot{\hat{\eta}}_k - \frac{\partial \alpha_{2,k}}{\partial \hat{S}_k}\dot{\hat{S}}_k - \frac{\partial \alpha_{2,k}}{\partial \hat{B}_{1,k}}\dot{\hat{B}}_{1,k} - \frac{\partial \alpha_{2,k}}{\partial \hat{b}_{1,k}}\dot{\hat{b}}_{1,k}.
\end{aligned} \tag{3.10}
$$

Construct Lyapunov function as follows:

$$
V_{3,k} = V_{2,k} + \frac{1}{2}z_{3,k}^2 + \frac{1}{2}\Lambda_2^{-1}\tilde{B}_{2,k}^2 + \frac{1}{2}\Upsilon_2^{-1}\tilde{b}_{2,k}^2, \tag{3.11}
$$

where both Λ_2 and Υ_2 are symmetric positive definite matrices. The time derivative of $V_{2,k}$

along systems (3.10) is given as:

$$
\begin{aligned}
\dot{V}_{3,k} &= \dot{V}_{2,k} + z_{3,k}\dot{z}_{3,k} - \tilde{B}_{2,k}\Lambda_2^{-1}\dot{\hat{B}}_{2,k} - \tilde{b}_{2,k}\Upsilon_2^{-1}\dot{\hat{b}}_{2,k} \\
&= \dot{V}_{2,k} + z_{3,k}(b_3(z_{4,k} + \alpha_{3,k} + y_{r,k}^{(3)}) + \eta^T\phi(t)\omega_{3,k} \\
&+ \delta^T(t)\omega_{3,k} - y_{r,k}^{(3)} - \frac{\partial\alpha_{2,k}}{\partial t} - \sum_{i=1}^{2}\frac{\partial\alpha_{2,k}}{\partial x_{i,k}}b_i x_{i+1,k} \\
&- \frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k - \frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k - \frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}\dot{\hat{B}}_{1,k} - \frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}\dot{\hat{b}}_{1,k}) \\
&- \tilde{B}_{2,k}\Lambda_2^{-1}\dot{\hat{B}}_{2,k} - \tilde{b}_{2,k}\Upsilon_2^{-1}\dot{\hat{b}}_{2,k} \\
&\leq -\sum_{i=1}^{2}(c_i - \frac{1}{4})z_{i,k}^2 + \sum_{i=1}^{2}(b_i N(\zeta_{i,k}(t)) + 1)\dot{\zeta}_{i,k}(t) \\
&+ \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{2,k} - \dot{\hat{\eta}}_k) + \frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}(\tau_{2,k} - \dot{\hat{\eta}}_k)z_{2,k} \\
&+ \tilde{S}_k\Gamma_2^{-1}(\nu_{2,k} - \dot{\hat{S}}_k) + \frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}(\nu_{2,k} - \dot{\hat{S}}_k)z_{2,k} \\
&+ \tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2 - \dot{\hat{B}}_{1,k}) + \tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k} - \dot{\hat{b}}_{1,k}) \\
&+ \frac{2}{4}\Delta_k + B_2 z_{3,k}^2 + b_3 z_{3,k}z_{4,k} + b_3(\alpha_{3,k} + y_{r,k}^{(3)})z_{3,k} + \delta^T(t)\omega_{3,k}z_{3,k} \\
&+ z_{3,k}(\eta^T\phi(t)\omega_{3,k} - y_{r,k}^{(3)} - \frac{\partial\alpha_{2,k}}{\partial t} - \sum_{i=1}^{2}\frac{\partial\alpha_{2,k}}{\partial x_{i,k}}b_i x_{i+1,k} \\
&- \frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k - \frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k - \frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}\dot{\hat{B}}_{1,k} - \frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}\dot{\hat{b}}_{1,k}) \\
&- \tilde{B}_{2,k}\Lambda_2^{-1}\dot{\hat{B}}_{2,k} - \tilde{b}_{2,k}\Upsilon_2^{-1}\dot{\hat{b}}_{2,k} \\
&\leq -\sum_{i=1}^{2}(c_i - \frac{1}{4})z_{i,k}^2 + \sum_{i=1}^{2}(b_i N(\zeta_{i,k}(t)) + 1)\dot{\zeta}_{i,k}(t) \\
&+ \tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{3,k} - \dot{\hat{\eta}}_k) + \frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}(\tau_{3,k} - \dot{\hat{\eta}}_k)z_{2,k} + \frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}(\tau_{3,k} - \dot{\hat{\eta}}_k)z_{3,k} \\
&+ \tilde{S}_k\Gamma_2^{-1}(\nu_{2,k} - \dot{\hat{S}}_k) + \frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}(\nu_{2,k} - \dot{\hat{S}}_k)z_{2,k} \\
&+ \tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2 - \dot{\hat{B}}_{1,k}) + \frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}(\Lambda_1 z_{2,k}^2 - \dot{\hat{B}}_{1,k})z_{3,k} \\
&+ \tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k} + \frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k}) - \dot{\hat{b}}_{1,k}) \\
&+ \frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k} + \frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k}) - \dot{\hat{b}}_{1,k})z_{3,k} \\
&+ \frac{2}{4}\Delta_k + b_3 z_{3,k}z_{4,k} + b_3(\alpha_{3,k} + y_{r,k}^{(3)})z_{3,k} + s\bar{\varphi}_{3,k}|z_{3,k}| \\
&+ z_{3,k}(\hat{\eta}^T\phi(t)\omega_{3,k} + \hat{B}_{2,k}z_{3,k} - y_{r,k}^{(3)} - \frac{\partial\alpha_{2,k}}{\partial t} - \sum_{i=1}^{2}\frac{\partial\alpha_{2,k}}{\partial x_{i,k}}\hat{b}_{i,k}x_{i+1,k} \\
&- \frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}\tau_{3,k} - \frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\Gamma_1\phi(t)\omega_{3,k}z_{2,k} - \frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k - \frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}\Lambda_1 z_{2,k}^2 \\
&- \frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k} + \frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k}))),
\end{aligned}
$$

$$
\begin{aligned}
&+\tilde{B}_{2,k}\Lambda_2^{-1}(\Lambda_2 z_{3,k}^2-\dot{\hat{B}}_{2,k})+\tilde{b}_{2,k}\Upsilon_2^{-1}(-\Upsilon_2\frac{\partial\alpha_{2,k}}{\partial x_{2,k}}x_{3,k}z_{3,k}-\dot{\hat{b}}_{2,k})\\
&\leq-\sum_{i=1}^{2}(c_i-\frac{1}{4})z_{i,k}^2+\sum_{i=1}^{2}(b_iN(\zeta_{i,k}(t))+1)\dot{\zeta}_{i,k}(t)\\
&+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{3,k}-\dot{\hat{\eta}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}(\tau_{3,k}-\dot{\hat{\eta}}_k)z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}(\tau_{3,k}-\dot{\hat{\eta}}_k)z_{3,k}\\
&+\tilde{S}_k\Gamma_2^{-1}(\nu_{2,k}-\dot{\hat{S}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}(\nu_{2,k}-\dot{\hat{S}}_k)z_{2,k}\\
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})z_{3,k}\\
&+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})-\dot{\hat{b}}_{1,k})\\
&+\frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})-\dot{\hat{b}}_{1,k})z_{3,k}\\
&+\frac{3}{4}\Delta_k+b_3z_{3,k}z_{4,k}+b_3(\alpha_{3,k}+y_{r,k}^{(3)})z_{3,k}+\frac{1}{\Delta_k}S\bar{\varphi}_{3,k}^2z_{3,k}^2\\
&+z_{3,k}(\hat{\eta}^T\phi(t)\omega_{3,k}+\hat{B}_{2,k}z_{3,k}-y_{r,k}^{(3)}-\frac{\partial\alpha_{2,k}}{\partial t}-\sum_{i=1}^{2}\frac{\partial\alpha_{2,k}}{\partial x_{i,k}}\hat{b}_{i,k}x_{i+1,k}\\
&-\frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}\tau_{3,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\Gamma_1\phi(t)\omega_{3,k}z_{2,k}-\frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k-\frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}\Lambda_1z_{2,k}^2\\
&-\frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})))\\
&+\tilde{B}_{2,k}\Lambda_2^{-1}(\Lambda_2 z_{3,k}^2-\dot{\hat{B}}_{2,k})+\tilde{b}_{2,k}\Upsilon_2^{-1}(-\Upsilon_2\frac{\partial\alpha_{2,k}}{\partial x_{2,k}}x_{3,k}z_{3,k}-\dot{\hat{b}}_{2,k})\\
&\leq-\sum_{i=1}^{2}(c_i-\frac{1}{4})z_{i,k}^2+\sum_{i=1}^{2}(b_iN(\zeta_{i,k}(t))+1)\dot{\zeta}_{i,k}(t)\\
&+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{3,k}-\dot{\hat{\eta}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}(\tau_{3,k}-\dot{\hat{\eta}}_k)z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}(\tau_{3,k}-\dot{\hat{\eta}}_k)z_{3,k}\\
&+\tilde{S}_k\Gamma_2^{-1}(\nu_{3,k}-\dot{\hat{S}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}(\nu_{3,k}-\dot{\hat{S}}_k)z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}(\nu_{3,k}-\dot{\hat{S}}_k)z_{3,k}\\
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})z_{3,k}\\
&+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})-\dot{\hat{b}}_{1,k})\\
&+\frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})-\dot{\hat{b}}_{1,k})z_{3,k}\\
&+\frac{3}{4}\Delta_k+b_3z_{3,k}z_{4,k}+b_3(\alpha_{3,k}+y_{r,k}^{(3)})z_{3,k}\\
&+z_{3,k}(\hat{\eta}^T\phi(t)\omega_{3,k}+\frac{1}{\Delta_k}\hat{S}_k\bar{\varphi}_{3,k}^2z_{3,k}+\hat{B}_{2,k}z_{3,k}-y_{r,k}^{(3)}-\frac{\partial\alpha_{2,k}}{\partial t}\\
&-\sum_{i=1}^{2}\frac{\partial\alpha_{2,k}}{\partial x_{i,k}}\hat{b}_{i,k}x_{i+1,k}-\frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}\tau_{3,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\Gamma_1\phi(t)\omega_{3,k}z_{2,k},
\end{aligned}
\tag{3.12}
$$

$$
\begin{aligned}
&-\frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}\nu_{3,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{3,k}^2 z_{2,k}z_{3,k}-\frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}\Lambda_1 z_{2,k}^2\\
&-\frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})))\\
&+\tilde{B}_{2,k}\Lambda_2^{-1}(\Lambda_2 z_{3,k}^2-\dot{\hat{B}}_{2,k})+\tilde{b}_{2,k}\Upsilon_2^{-1}(-\Upsilon_2\frac{\partial\alpha_{2,k}}{\partial x_{2,k}}x_{3,k}z_{3,k}-\dot{\hat{b}}_{2,k}). \qquad (3.13)
\end{aligned}
$$

Take virtual control as

$$\alpha_{3,k}=N(\zeta_{3,k}(t))\psi_{3,k}(t)-y_{r,k}^{(3)}, \qquad (3.14)$$

$$
\begin{aligned}
\psi_{3,k}(t)&=c_3 z_{3,k}+\hat{\eta}^T\phi(t)\omega_{3,k}+\frac{1}{\Delta_k}\hat{S}_k\bar{\varphi}_{3,k}^2 z_{3,k}+\hat{B}_{2,k}z_{3,k}-y_{r,k}^{(3)}\\
&-\frac{\partial\alpha_{2,k}}{\partial t}-\sum_{i=1}^{2}\frac{\partial\alpha_{2,k}}{\partial x_{i,k}}\hat{b}_{i,k}x_{i+1,k}-\frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}\tau_{3,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}\Gamma_1\phi(t)\omega_{3,k}z_{2,k}\\
&-\frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}\nu_{3,k}-\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}\Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{3,k}^2 z_{2,k}z_{3,k}\\
&-\frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}\Lambda_1 z_{2,k}^2-\frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})),\dot{\zeta}_{3,k}(t)\\
&=\psi_{3,k}(t)z_{3,k}, \qquad (3.15)
\end{aligned}
$$

where $c_3>\frac{1}{4}$ is a constant, where $\hat{B}_{2,k}$, $\hat{b}_{2,k}$ are the estimation of parameters B_2, b_2, respectively. Both $\tilde{B}_{2,k}=B_2-\hat{B}_{2,k}$ and $\tilde{b}_{2,k}=b_2-\hat{b}_{2,k}$ are parameter estimation errors.
Then equation (3.13) can be rewrite as following:

$$
\begin{aligned}
\dot{V}_{3,k}\le&-\sum_{i=1}^{2}(c_i-\frac{1}{4})z_{i,k}^2+\sum_{i=1}^{2}(b_i N(\zeta_{i,k}(t))+1)\dot{\zeta}_{i,k}(t)\\
&+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{3,k}-\dot{\hat{\eta}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}(\tau_{3,k}-\dot{\hat{\eta}}_k)z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}(\tau_{3,k}-\dot{\hat{\eta}}_k)z_{3,k}\\
&+\tilde{S}_k\Gamma_2^{-1}(\nu_{3,k}-\dot{\hat{S}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}(\nu_{3,k}-\dot{\hat{S}}_k)z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}(\nu_{3,k}-\dot{\hat{S}}_k)z_{3,k}\\
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})z_{3,k}\\
&+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})-\dot{\hat{b}}_{1,k})\\
&+\frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})-\dot{\hat{b}}_{1,k})z_{3,k}\\
&+\tilde{B}_{2,k}\Lambda_2^{-1}(\Lambda_2 z_{3,k}^2-\dot{\hat{B}}_{2,k})+\tilde{b}_{2,k}\Upsilon_2^{-1}(-\Upsilon_2\frac{\partial\alpha_{2,k}}{\partial x_{2,k}}x_{3,k}z_{3,k}-\dot{\hat{b}}_{2,k})\\
&+\frac{3}{4}\Delta_k+b_3 z_{3,k}z_{4,k}-c_3 z_{3,k}^2+(b_3 N(\zeta_{3,k}(t))+1)\dot{\zeta}_{3,k}(t)\\
\le&-\sum_{i=1}^{3}(c_i-\frac{1}{4})z_{i,k}^2+\sum_{i=1}^{3}(b_i N(\zeta_{i,k}(t))+1)\dot{\zeta}_{i,k}(t)\\
&+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{3,k}-\dot{\hat{\eta}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{\eta}_k}(\tau_{3,k}-\dot{\hat{\eta}}_k)z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial\hat{\eta}_k}(\tau_{3,k}-\dot{\hat{\eta}}_k)z_{3,k}, \qquad (3.16)
\end{aligned}
$$

$$
\begin{aligned}
&+\tilde{S}_k\Gamma_2^{-1}(\nu_{3,k}-\dot{\hat{S}}_k)+\frac{\partial\alpha_{1,k}}{\partial\hat{S}_k}(\nu_{3,k}-\dot{\hat{S}}_k)z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial\hat{S}_k}(\nu_{3,k}-\dot{\hat{S}}_k)z_{3,k}\\
&+\tilde{B}_{1,k}\Lambda_1^{-1}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})+\frac{\partial\alpha_{2,k}}{\partial\hat{B}_{1,k}}(\Lambda_1 z_{2,k}^2-\dot{\hat{B}}_{1,k})z_{3,k}\\
&+\tilde{b}_{1,k}\Upsilon_1^{-1}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})-\dot{\hat{b}}_{1,k})\\
&+\frac{\partial\alpha_{2,k}}{\partial\hat{b}_{1,k}}(-\Upsilon_1(\frac{\partial\alpha_{1,k}}{\partial x_{1,k}}x_{2,k}z_{2,k}+\frac{\partial\alpha_{2,k}}{\partial x_{1,k}}x_{2,k}z_{3,k})-\dot{\hat{b}}_{1,k})z_{3,k}\\
&+\tilde{B}_{2,k}\Lambda_2^{-1}(\Lambda_2 z_{3,k}^2-\dot{\hat{B}}_{2,k})+\tilde{b}_{2,k}\Upsilon_2^{-1}(-\Upsilon_2\frac{\partial\alpha_{2,k}}{\partial x_{2,k}}x_{3,k}z_{3,k}-\dot{\hat{b}}_{2,k})+\frac{3}{4}\Delta_k+B_3 z_{4,k}^2.
\end{aligned}
\tag{3.17}
$$

Step i ($4 \le i \le n-1$). Let $\omega_{i,k} = \varphi_{i,k} - \Sigma_{j=1}^{i-1}\frac{\partial\alpha_{i-1,k}}{\partial x_{j,k}}\varphi_{j,k}$, because $\varphi_{1,k},\cdots,\varphi_{i,k}$ are known smooth functions, there exist smooth function $\bar{\varphi}_{i,k}(x_{1,k},\cdots,x_{i,k},\hat{\eta}_k,\hat{S}_k) > 0$ such that $|\delta^T(t)(\varphi_{i,k} - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1,k}}{\partial x_{j,k}}\varphi_{j,k})| \le \|\delta^T(t)\|\bar{\varphi}_{i,k} \le s\bar{\varphi}_{i,k}$. Denote $B_i = b_i^2$, $z_{i+1,k} = x_{i+1,k} - \alpha_{i,k} - y_r^{(i)}$, $\tau_{i,k} = \tau_{i-1,k} + \Gamma_1\phi(t)\omega_{i,k}z_{i,k}$, $\nu_{i,k} = \nu_{i-1,k} + \Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{i,k}^2 z_{i,k}^2$ then the time derivative of $z_{i,k}$ is given as following:

$$
\begin{aligned}
\dot{z}_{i,k} &= b_i(z_{i+1,k}+\alpha_{i,k}+y_{r,k}^{(i)})+\eta^T\phi(t)\omega_{i,k}+\delta^T(t)\omega_{i,k}-\frac{\partial\alpha_{i-1,k}}{\partial t}\\
&-\Sigma_{j=1}^{i-1}\frac{\partial\alpha_{i-1,k}}{\partial x_{j,k}}b_j x_{j+1,k}-\frac{\partial\alpha_{i-1,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k-\frac{\partial\alpha_{i-1,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k\\
&-\sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1,k}}{\partial\hat{B}_{j,k}}\dot{\hat{B}}_{j,k}-\sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1,k}}{\partial\hat{b}_{j,k}}\dot{\hat{b}}_{j,k}.
\end{aligned}
\tag{3.18}
$$

Construct Lyapunov function as follows:

$$
V_{i,k} = V_{i-1,k}+\frac{1}{2}z_{i,k}^2+\frac{1}{2}\Lambda_{i-1}^{-1}\tilde{B}_{i-1,k}^2+\frac{1}{2}\Upsilon_{i-1}^{-1}\tilde{b}_{i-1,k}^2.
\tag{3.19}
$$

Derivated $V_{i,k}$ by (3.18) as follows:

$$
\begin{aligned}
\dot{V}_{i,k} \le &-\sum_{j=1}^{i-1}(c_j-\frac{1}{4})z_{j,k}^2+\sum_{j=1}^{i-1}(b_jN(\zeta_{j,k}(t))+1)\dot{\zeta}_{j,k}(t)+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{i,k}-\dot{\hat{\eta}}_k)\\
&+\sum_{j=1}^{i-1}\frac{\partial\alpha_{j,k}}{\partial\hat{\eta}_k}(\tau_{i,k}-\dot{\hat{\eta}}_k)z_{j+1,k}+\tilde{S}_k\Gamma_2^{-1}(\nu_{i,k}-\dot{\hat{S}}_k)\\
&+\sum_{j=1}^{i-1}\frac{\partial\alpha_{j,k}}{\partial\hat{S}_k}(\nu_{i,k}-\dot{\hat{S}}_k)z_{j+1,k}+\sum_{j=1}^{i-1}\tilde{B}_{j,k}\Lambda_j^{-1}(\Lambda_j z_{j+1,k}^2-\dot{\hat{B}}_{j,k})\\
&+\sum_{l=1}^{i-2}\sum_{j=l+1}^{i-1}\frac{\partial\alpha_{j,k}}{\partial\hat{B}_{l,k}}(\Lambda_l z_{l+1,k}^2-\dot{\hat{B}}_{l,k})z_{j+1,k}\\
&+\sum_{j=1}^{i-1}\tilde{b}_{j,k}\Upsilon_j^{-1}(-\Upsilon_j\sum_{l=j}^{i-1}\frac{\partial\alpha_{l,k}}{\partial x_{j,k}}x_{j+1,k}z_{l+1,k}-\dot{\hat{b}}_{j,k}),
\end{aligned}
\tag{3.20}
$$

$$
\begin{aligned}
&+\sum_{l=1}^{i-2}\sum_{j=l+1}^{i-1}\frac{\partial\alpha_{j,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\sum_{u=l}^{i-1}\frac{\partial\alpha_{u,k}}{\partial x_{l,k}}x_{l+1,k}z_{u+1,k}-\dot{\hat{b}}_{l,k})z_{j+1,k}\\
&+\frac{i}{4}\Delta_k+b_iz_{i,k}z_{i+1,k}+b_i(\alpha_{i,k}+y_{r,k}^{(i)})z_{i,k}\\
&+z_{i,k}(\hat{\eta}^T\phi(t)\omega_{i,k}+\frac{1}{\Delta_k}\hat{S}_k\bar{\varphi}_{i,k}^2z_{i,k}+\hat{B}_{i-1,k}z_{i,k}-y_{r,k}^{(i)}\\
&-\frac{\partial\alpha_{i-1,k}}{\partial t}-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1,k}}{\partial x_{j,k}}\hat{b}_{j,k}x_{j+1,k}\\
&-\frac{\partial\alpha_{i-1,k}}{\partial\hat{\eta}_k}\tau_{i,k}-\Sigma_{j=1}^{i-2}\frac{\partial\alpha_{j,k}}{\partial\hat{\eta}_k}\Gamma_1\phi(t)\omega_{i,k}z_{j+1,k}-\frac{\partial\alpha_{i-1,k}}{\partial\hat{S}_k}\nu_{i,k}\\
&-\Sigma_{j=1}^{i-2}\frac{\partial\alpha_{j,k}}{\partial\hat{s}_k}\Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{i,k}^2z_{i,k}z_{j+1,k}-\sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1,k}}{\partial\hat{B}_{j,k}}\Lambda_jz_{j+1,k}^2\\
&-\sum_{l=1}^{i-2}\frac{\partial\alpha_{i-1,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\sum_{j=l}^{i-1}\frac{\partial\alpha_{j,k}}{\partial x_{l,k}}x_{l+1,k}z_{j+1,k})\\
&-\sum_{l=1}^{i-3}\sum_{j=l+1}^{i-2}\frac{\partial\alpha_{j,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\frac{\partial\alpha_{i-1,k}}{\partial x_{l,k}}x_{l+1,k}z_{j+1,k})).
\end{aligned}\tag{3.21}
$$

Take virtual control as

$$
\alpha_{i,k}=N(\zeta_{i,k}(t))\psi_{i,k}(t)-y_{r,k}^{(i)},\tag{3.22}
$$

$$
\begin{aligned}
\psi_{i,k}(t)=\;&c_iz_{i,k}+\hat{\eta}^T\phi(t)\omega_{i,k}+\frac{1}{\Delta_k}\hat{S}_k\bar{\varphi}_{i,k}^2z_{i,k}+\hat{B}_{i-1,k}z_{i,k}-y_{r,k}^{(i)}\\
&-\frac{\partial\alpha_{i-1,k}}{\partial t}-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1,k}}{\partial x_{j,k}}\hat{b}_{j,k}x_{j+1,k}-\frac{\partial\alpha_{i-1,k}}{\partial\hat{\eta}_k}\tau_{i,k}\\
&-\Sigma_{j=1}^{i-2}\frac{\partial\alpha_{j,k}}{\partial\hat{\eta}_k}\Gamma_1\phi(t)\omega_{i,k}z_{j+1,k}-\frac{\partial\alpha_{i-1,k}}{\partial\hat{S}_k}\nu_{i,k}\\
&-\Sigma_{j=1}^{i-2}\frac{\partial\alpha_{j,k}}{\partial\hat{s}_k}\Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{i,k}^2z_{i,k}z_{j+1,k}-\sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1,k}}{\partial\hat{B}_{j,k}}\Lambda_jz_{j+1,k}^2\\
&-\sum_{l=1}^{i-2}\frac{\partial\alpha_{i-1,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\sum_{j=l}^{i-1}\frac{\partial\alpha_{j,k}}{\partial x_{l,k}}x_{l+1,k}z_{j+1,k})\\
&-\sum_{l=1}^{i-3}\sum_{j=l+1}^{i-2}\frac{\partial\alpha_{j,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\frac{\partial\alpha_{i-1,k}}{\partial x_{l,k}}x_{l+1,k}z_{j+1,k}),\dot{\zeta}_{i,k}(t)=\psi_{i,k}(t)z_{i,k},
\end{aligned}\tag{3.23}
$$

where $c_i>\frac{1}{4}$ is a constant, where $\hat{B}_{i-1,k}$, $\hat{b}_{i-1,k}$ are the estimation of parameters B_{i-1}, b_{i-1}, respectively. Both $\tilde{B}_{i-1,k}=B_{i-1}-\hat{B}_{i-1,k}$ and $\tilde{b}_{i-1,k}=b_{i-1}-\hat{b}_{i-1,k}$ are parameter estimation errors. Then equation (3.21) can be rewrite as following:

$$
\begin{aligned}
\dot{V}_{i,k}\leq\;&-\sum_{j=1}^{i}(c_j-\frac{1}{4})z_{j,k}^2+\sum_{j=1}^{i}(b_jN(\zeta_{j,k}(t))+1)\dot{\zeta}_{j,k}(t)+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{i,k}-\dot{\hat{\eta}}_k)\\
&+\sum_{j=1}^{i-1}\frac{\partial\alpha_{j,k}}{\partial\hat{\eta}_k}(\tau_{i,k}-\dot{\hat{\eta}}_k)z_{j+1,k}+\tilde{S}_k\Gamma_2^{-1}(\nu_{i,k}-\dot{\hat{S}}_k),
\end{aligned}\tag{3.24}
$$

$$+\sum_{j=1}^{i-1}\frac{\partial\alpha_{j,k}}{\partial\hat{S}_k}(\nu_{i,k}-\dot{\hat{S}}_k)z_{j+1,k}+\sum_{j=1}^{i-1}\tilde{B}_{j,k}\Lambda_j^{-1}(\Lambda_j z_{j+1,k}^2-\dot{\hat{B}}_{j,k})$$
$$+\sum_{l=1}^{i-2}\sum_{j=l+1}^{i-1}\frac{\partial\alpha_{j,k}}{\partial\hat{B}_{l,k}}(\Lambda_l z_{l+1,k}^2-\dot{\hat{B}}_{l,k})z_{j+1,k}$$
$$+\sum_{j=1}^{i-1}\tilde{b}_{j,k}\Upsilon_j^{-1}(-\Upsilon_j\sum_{l=j}^{i-1}\frac{\partial\alpha_{l,k}}{\partial x_{j,k}}x_{j+1,k}z_{l+1,k}-\dot{\hat{b}}_{j,k})$$
$$+\sum_{l=1}^{i-2}\sum_{j=l+1}^{i-1}\frac{\partial\alpha_{j,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\sum_{u=l}^{i-1}\frac{\partial\alpha_{u,k}}{\partial x_{l,k}}x_{l+1,k}z_{u+1,k}-\dot{\hat{b}}_{l,k})z_{j+1,k}$$
$$+\frac{i}{4}\Delta_k+B_i z_{i+1,k}^2. \tag{3.25}$$

Step n. Let $\omega_{n,k}=\varphi_{n,k}-\Sigma_{j=1}^{n-1}\frac{\partial\alpha_{n-1,k}}{\partial x_{j,k}}\varphi_{j,k}$, because $\varphi_{1,k},\cdots,\varphi_{n,k}$ are known, then have $\bar{\varphi}_{n,k}$ $(x_{1,k},\cdots,x_{n,k},\hat{\eta}_k,\hat{S}_k)>0$ satisfied

$$|\delta^T(t)(\varphi_{n,k}-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1,k}}{\partial x_{j,k}}\varphi_{j,k})|\le\|\delta^T(t)\|\bar{\varphi}_{n,k}\le s\bar{\varphi}_{n,k}. \tag{3.26}$$

Take

$$\tau_{n,k}=\tau_{n-1,k}+\Gamma_1\phi(t)\omega_{n,k}z_{n,k},\nu_{n,k}=\nu_{n-1,k}+\Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{n,k}^2z_{n,k}^2, \tag{3.27}$$

then derivated $z_{n,k}$ as following:

$$\dot{z}_{n,k}=bu_k-y_{r,k}^{(n)}+\eta^T\phi(t)\omega_{n,k}+\delta^T(t)\omega_{n,k}-\frac{\partial\alpha_{n-1,k}}{\partial t}$$
$$-\Sigma_{j=1}^{n-1}\frac{\partial\alpha_{n-1,k}}{\partial x_{j,k}}b_jx_{j+1,k}-\frac{\partial\alpha_{n-1,k}}{\partial\hat{\eta}_k}\dot{\hat{\eta}}_k-\frac{\partial\alpha_{n-1,k}}{\partial\hat{S}_k}\dot{\hat{S}}_k$$
$$-\sum_{j=1}^{n-2}\frac{\partial\alpha_{n-1,k}}{\partial\hat{B}_{j,k}}\dot{\hat{B}}_{j,k}-\sum_{j=1}^{n-2}\frac{\partial\alpha_{n-1,k}}{\partial\hat{b}_{j,k}}\dot{\hat{b}}_{j,k}. \tag{3.28}$$

Construct Lyapunov function as follows:

$$V_{n,k}=V_{n-1,k}+\frac{1}{2}z_{n,k}^2+\frac{1}{2}\Lambda_{n-1}^{-1}\tilde{B}_{n-1,k}^2+\frac{1}{2}\Upsilon_{n-1}^{-1}\tilde{b}_{n-1,k}^2, \tag{3.29}$$

then

$$\dot{V}_{n,k}\le-\sum_{j=1}^{n-1}(c_j-\frac{1}{4})z_{j,k}^2+\sum_{j=1}^{n-1}(b_jN(\zeta_{j,k}(t))+1)\dot{\zeta}_{j,k}(t)$$
$$+\tilde{\eta}_k^T\Gamma_1^{-1}(\tau_{n,k}-\dot{\hat{\eta}}_k)+\sum_{j=1}^{n-1}\frac{\partial\alpha_{j,k}}{\partial\hat{\eta}_k}(\tau_{n,k}-\dot{\hat{\eta}}_k)z_{j+1,k}$$
$$+\tilde{S}_k\Gamma_2^{-1}(\nu_{n,k}-\dot{\hat{S}}_k)+\sum_{j=1}^{n-1}\frac{\partial\alpha_{j,k}}{\partial\hat{S}_k}(\nu_{n,k}-\dot{\hat{S}}_k)z_{j+1,k}$$
$$+\sum_{j=1}^{n-1}\tilde{B}_{j,k}\Lambda_j^{-1}(\Lambda_jz_{j+1,k}^2-\dot{\hat{B}}_{j,k}),$$

$$
\begin{aligned}
&+\sum_{l=1}^{n-2}\sum_{j=l+1}^{n-1}\frac{\partial\alpha_{j,k}}{\partial\hat{B}_{l,k}}(\Lambda_l z_{l+1,k}^2-\dot{\hat{B}}_{l,k})z_{j+1,k}\\
&+\sum_{j=1}^{n-1}\tilde{b}_{j,k}\Upsilon_j^{-1}(-\Upsilon_j\sum_{l=j}^{n-1}\frac{\partial\alpha_{l,k}}{\partial x_{j,k}}x_{j+1,k}z_{l+1,k}-\dot{\hat{b}}_{j,k})\\
&+\sum_{l=1}^{n-2}\sum_{j=l+1}^{n-1}\frac{\partial\alpha_{j,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\sum_{u=l}^{n-1}\frac{\partial\alpha_{u,k}}{\partial x_{l,k}}x_{l+1,k}z_{u+1,k}-\dot{\hat{b}}_{l,k})z_{j+1,k}\\
&+\frac{n}{4}\Delta_k+b_n u_k z_{n,k}\\
&+z_{n,k}(\hat{\eta}^T\phi(t)\omega_{n,k}+\frac{1}{\Delta_k}\hat{S}_k\bar{\varphi}_{n,k}^2 z_{n,k}+\hat{B}_{n-1,k}z_{n,k}\\
&-y_{r,k}^{(n)}-\frac{\partial\alpha_{n-1,k}}{\partial t}-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1,k}}{\partial x_{j,k}}\hat{b}_{j,k}x_{j+1,k}\\
&-\frac{\partial\alpha_{n-1,k}}{\partial\hat{\eta}_k}\tau_{n,k}-\Sigma_{j=1}^{n-2}\frac{\partial\alpha_{j,k}}{\partial\hat{\eta}_k}\Gamma_1\phi(t)\omega_{n,k}z_{j+1,k}-\frac{\partial\alpha_{n-1,k}}{\partial\hat{S}_k}\nu_{n,k}\\
&-\Sigma_{j=1}^{n-2}\frac{\partial\alpha_{j,k}}{\partial\hat{s}_k}\Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{n,k}^2 z_{n,k}z_{j+1,k}-\sum_{j=1}^{n-2}\frac{\partial\alpha_{n-1,k}}{\partial\hat{B}_{j,k}}\Lambda_j z_{j+1,k}^2\\
&-\sum_{l=1}^{n-2}\frac{\partial\alpha_{n-1,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\sum_{j=l}^{n-1}\frac{\partial\alpha_{j,k}}{\partial x_{l,k}}x_{l+1,k}z_{j+1,k})\\
&-\sum_{l=1}^{n-3}\sum_{j=l+1}^{n-2}\frac{\partial\alpha_{j,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\frac{\partial\alpha_{n-1,k}}{\partial x_{l,k}}x_{l+1,k}z_{j+1,k})).
\end{aligned}
\tag{3.30}
$$

Controller and adaptive laws are designed as follows:

$$u_k=N(\zeta_{n,k}(t))\psi_{n,k}(t),\tag{3.31}$$

$$
\begin{aligned}
\psi_{n,k}(t)=&(c_n-\frac{1}{4})z_{n,k}+\hat{\eta}^T\phi(t)\omega_{n,k}+\frac{1}{\Delta_k}\hat{S}_k\bar{\varphi}_{n,k}^2 z_{n,k}\\
&+\hat{B}_{n-1,k}z_{n,k}-y_{r,k}^{(n)}-\frac{\partial\alpha_{n-1,k}}{\partial t}-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1,k}}{\partial x_{j,k}}\hat{b}_{j,k}x_{j+1,k}\\
&-\frac{\partial\alpha_{n-1,k}}{\partial\hat{\eta}_k}\tau_{n,k}-\Sigma_{j=1}^{n-2}\frac{\partial\alpha_{j,k}}{\partial\hat{\eta}_k}\Gamma_1\phi(t)\omega_{n,k}z_{j+1,k}-\frac{\partial\alpha_{n-1,k}}{\partial\hat{S}_k}\nu_{n,k}\\
&-\Sigma_{j=1}^{n-2}\frac{\partial\alpha_{j,k}}{\partial\hat{s}_k}\Gamma_2\frac{1}{\Delta_k}\bar{\varphi}_{n,k}^2 z_{n,k}z_{j+1,k}-\sum_{j=1}^{n-2}\frac{\partial\alpha_{n-1,k}}{\partial\hat{B}_{j,k}}\Lambda_j z_{j+1,k}^2\\
&-\sum_{l=1}^{n-2}\frac{\partial\alpha_{n-1,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\sum_{j=l}^{n-1}\frac{\partial\alpha_{j,k}}{\partial x_{l,k}}x_{l+1,k}z_{j+1,k})\\
&-\sum_{l=1}^{n-3}\sum_{j=l+1}^{n-2}\frac{\partial\alpha_{j,k}}{\partial\hat{b}_{l,k}}(-\Upsilon_l\frac{\partial\alpha_{n-1,k}}{\partial x_{l,k}}x_{l+1,k}z_{j+1,k}),
\end{aligned}
\tag{3.32}
$$

$$\dot{\zeta}_{n,k}(t)=z_{n,k}\psi_{n,k}(t),\dot{\hat{\eta}}_k=\tau_{n,k},\tag{3.33}$$

$$\dot{\hat{S}}_k=\nu_{n,k},\tag{3.34}$$

$$\dot{\hat{B}}_{i,k} = \Lambda_i z_{i+1,k}^2,\ i = 1, \cdots, n-1, \tag{3.35}$$

$$\dot{\hat{b}}_{i,k} = -\Upsilon_i \sum_{l=i}^{n-1} \frac{\partial \alpha_{l,k}}{\partial x_{i,k}} x_{i+1,k} z_{l+1,k},\ i = 1, \cdots, n-1. \tag{3.36}$$

Derivated $V_{n,k}$ by (3.28), and substitute eqs.((3.31)-(3.36)) into eq. (3.30), then

$$\dot{V}_{n,k} \leq -\Sigma_{j=1}^{n}(c_j - \frac{1}{4})z_{j,k}^2 + \sum_{j=1}^{n}(b_j N(\zeta_{j,k}(t)) + 1)\dot{\zeta}_{j,k}(t) + \frac{n}{4}\Delta_k. \tag{3.37}$$

3.2 Stability and convergence analysis

Under the condition that hypothesis 1 is satisfied, the conclusions of this paper are given as follows:

Assumption 1 For any k, when $t = 0$, $x_{1,k}(0) = y_{r,k}(0)$; $x_{i+1,k}(0) = \alpha_{i,k}(0) - y_{r,k}^{(i)}(0)$, $i = 1, \cdots, n-1$; $\hat{\eta}_k(0) = \hat{\eta}_{k-1}(T)$; $\hat{S}_k(0) = \hat{S}_{k-1}(T)$; $\hat{B}_{i,k}(0) = \hat{B}_{i,k-1}(T)$, $i = 1, \cdots, n-1$; $\hat{b}_{i,k}(0) = \hat{b}_{i,k-1}(T)$, $i = 1, \cdots, n-1$; $\dot{\zeta}_{i,k}(0) = \dot{\zeta}_{i,k-1}(T)$, $i = 1, \cdots, n$; $\zeta_{i,k}(0) = \zeta_{i,k-1}(T)$, $\zeta_{i,0}(0) = 0$, $i = 1, \cdots, n$.

Theorem 3.1. *Design controllers (3.31)-(3.32) and adaptive laws (3.33)-(3.36) For time-varying parameter system (1), all signals of closed loop system are bounded on $[0, T]$, and*

$$lim_{k\to\infty} z_{j,k}(t) = 0,\ j = 1, 2, \cdots, n \tag{3.38}$$

Proof. By the assumption 1, $\|z_k(0)\|^2 = 0 \leq \|z_k(T)\|^2$. Denote $V_{n,k}(0) = V_{n,k}(z_k(0), \hat{\eta}_k(0), \hat{S}_k(0), \hat{B}_{i,k}(0), \hat{b}_{i,k}(0))$, By eq.(13), we obtain

$$V_{n,k}(z_k(0), \hat{\eta}_k(T), \hat{S}_k(T), \hat{B}_{i,k}(T), \hat{b}_{i,k}(T)) \leq V_{n,k}(0) + \int_0^T \dot{V}_{n,k} dt, i = 1, \cdots, n-1 \tag{3.39}$$

Substitute eq.(3.37) into eq. (3.39), then

$$\begin{aligned} V_{n,k}(z_k(0), \hat{\eta}_k(T), \hat{S}_k(T), \hat{B}_{i,k}(T), \hat{b}_{i,k}(T)) \leq\ & V_{n,1}(0) - \Sigma_{i=1}^{k}\Sigma_{j=1}^{n} \\ & \int_0^T (c_j - \frac{1}{4})(z_{j,i})^2 dt \\ & + \frac{n}{4}T(\Sigma_{i=1}^{k}\Delta_i) + \Sigma_{i=1}^{k}\Sigma_{j=1}^{n} \\ & \int_0^T (b_j N(\zeta_{j,i}(t)) + 1)\dot{\zeta}_{j,i}(t) dt. \end{aligned} \tag{3.40}$$

Denote $\dot{\zeta}_i(t + (k-1)T) \triangleq \dot{\zeta}_{i,k}(t)$, and $\zeta_i(t + (k-1)T) = \zeta_{i,k}(t)$ for $t \in [0, T]$. According to assumption 1 and controllers (3.31)-(3.32), $\dot{\zeta}_i(t)$ is a continuous function and $\zeta_i(t)$ is a C^1

function for $\forall t \in [0, kT]$. So

$$\begin{aligned}
\Sigma_{j=1}^{k} \int_0^T (b_i N(\zeta_{i,j}) + 1)\dot{\zeta}_{i,j} d\tau = & \int_0^T (b_i N(\zeta_{i,1}) + 1)\dot{\zeta}_{i,1} d\tau \\
& + \int_0^T (b_i N(\zeta_{i,2}) + 1)\dot{\zeta}_{i,2} d\tau + \cdots + \\
& + \int_0^T (b_i N(\zeta_{i,k}) + 1)\dot{\zeta}_{i,k} d\tau \\
& = \int_0^T (b_i N(\zeta_i) + 1)\dot{\zeta}_i d\tau \\
& + \int_T^{2T} (b_i N(\zeta_i) + 1)\dot{\zeta}_i d\tau + \cdots \\
& + \int_{(k-1)T}^{kT} (b_i N(\zeta_i) + 1)\dot{\zeta}_i d\tau \\
& = \int_0^{kT} (b_i N(\zeta_i) + 1)\dot{\zeta}_i d\tau. \qquad (3.41)
\end{aligned}$$

For $B > 0$, we have $\int_0^{kT} (b_i N(\zeta_i) + 1)\dot{\zeta}_i d\tau \le B$. Denote

$$\begin{aligned}
V_0(k) = & V_{n,1}(z_1(0), \hat{\eta}_1(0), \hat{S}_1(0), \hat{B}_{i,1}(0), \hat{b}_{i,1}(0)) \\
& + n\frac{1}{4}T(\Sigma_{i=1}^{k}\Delta_i) + \Sigma_{i=1}^{k}\Sigma_{j=1}^{n} \int_0^T (b_j N(\zeta_{j,i}(t)) + 1)\dot{\zeta}_{j,i}(t) dt, \qquad (3.42)
\end{aligned}$$

then eq. (3.40) can be rewrite as

$$\Sigma_{i=1}^{k}\Sigma_{j=1}^{n} \int_0^T (c_j - \frac{1}{4})(z_{j,i})^2 dt \le V_0(k) - V_{n,k}(z_k(0), \hat{\eta}_k(T), \hat{S}_k(T), \hat{B}_{i,k}(T), \hat{b}_{i,k}(T)). \qquad (3.43)$$

We have $lim_{k\to\infty} V_0(k) \le V_{n,1} + nB + 2an\frac{1}{4}T$, then $V_0(k)$ is bounded, and $V_{n,k}(z_k(0), \hat{\eta}_k(T), \hat{S}_k(T)) \ge 0$, so

$$lim_{k\to\infty}\Sigma_{j=1}^{n} \int_0^T (c_j - \frac{1}{4})(z_{j,k})^2 dt = 0. \qquad (3.44)$$

From eq. (3.3), for $\forall k$, $V_{n,k}(t) = V_{n,k}(0) + \int_0^t \dot{V}_{n,k}(\tau) d\tau$, eq.(30) is substituted into above equation, then

$$\begin{aligned}
V_{n,k}(t) \le & V_{n,k}(0) - \Sigma_{j=1}^{n} \int_0^t (c_j - \frac{1}{4})(z_{j,k}(\tau))^2 d\tau \\
& + \Sigma_{j=1}^{n} \int_0^t (b_j N(\zeta_{j,k}(\tau)) + 1)\dot{\zeta}_{j,k}(\tau) d\tau + tn\frac{1}{4}\Delta_k. \qquad (3.45)
\end{aligned}$$

By eq.(3.44), $\Sigma_{j=1}^{n} \int_0^t (c_j - \frac{1}{4})(z_{j,k}(\tau))^2 d\tau$ is bounded. Δ_k is bounded, and $t \in [0, T]$, so $tn\frac{1}{4}\Delta_k$ is also bounded. By lemma 2, $\Sigma_{j=1}^{n} \int_0^t (b_j N(\zeta_{j,k}(\tau)) + 1)\dot{\zeta}_{j,k}(\tau) d\tau$ is bounded. And also $\hat{\eta}_k(0) = \hat{\eta}_{k-1}(T)$, $\hat{S}_k(0) = \hat{S}_{k-1}(T)$; $\hat{B}_{i,k}(0) = \hat{B}_{i,k-1}(T)$, $i = 1, \cdots, n-1$; $\hat{b}_{i,k}(0) = \hat{b}_{i,k-1}(T)$, by eq.(3.40), for any k, $V_{n,k}(0, \hat{\eta}_k(T), \hat{S}_k(T), \hat{B}_{i,k}(T), \hat{b}_{i,k}(T))$ is bounded, so $V_{n,k}$ $(0, \hat{\eta}_k(0), \hat{S}_k(0), \hat{B}_{i,k}(0), \hat{b}_{i,k}(0))$ $= V_{n,k-1}(0, \hat{\eta}_{k-1}(T), \hat{S}_{k-1}(T), \hat{B}_{i,k-1}(T), \hat{b}_{i,k-1}(T))$ is bounded, so for $\forall k$, $V_{n,k}(t)$ is bounded, then we have $x_{i,k}$, $\hat{\eta}_k(t)$, $\hat{S}_k(t)$, $\hat{B}_{i,k}(t)$ and $\hat{b}_{i,k}(t)$ are bounded. By eqs. (3.31)-(3.32), u_k is bounded. By (3.18), $\dot{z}_{i,k}$ is bounded, by $z_{i,k}$ is continuous uniformly, the formula (3.38) is established. □

4 An illustrative example

To demonstrate the effectiveness of the controller, consider the following nonlinear system:

$$\begin{aligned} \dot{x}_{1,k} &= b_1 x_{2,k} + \theta(t) x_{1,k}^2, \\ \dot{x}_{2,k} &= b_2 u_k(t), \\ y_k &= x_{1,k}, \end{aligned} \tag{4.1}$$

where $t \in$, $\theta(t)$ is uncertain time-varying parameter in finite time interval $[0,1]$. By Fourier series, then system (4.1) becomes

$$\begin{aligned} \dot{x}_{1,k} &= b_1 x_{2,k} + \eta^T \phi(t) x_{1,k}^2 + \delta^T(t) x_{1,k}^2, \\ \dot{x}_{2,k} &= b_2 u_k(t), \\ y_k &= x_{1,k}, \end{aligned} \tag{4.2}$$

where $x_{1,k}$ and $x_{2,k}$ are two states, $u_k(t)$ is the input of system. $\eta = [\eta_1, \eta_2, \eta_3, \eta_4, \eta_5]^T$, $\phi(t) = [1, sin(t), cos(t), sin(2t), cos(2t)]^T$, $\|\delta(t)\| \le s$, s is unknown . Take $\theta(t) = sin(2\pi t)$, $b_1 = -10$, $b_2 = 0.1$. Given the target $y_{r,k} = g_k sin(2t)$, for non-uniform target case, $g_k = -0.1$ for even k, and $g_k = 0.1$ for odd k.

Step 1. Let $z_{1,k} = x_{1,k} - y_{r,k}$, $z_{2,k} = x_{2,k} - \alpha_{1,k} - \dot{y}_{r,k}$, where $\alpha_{1,k} = N(\zeta_{1,k}(t))\psi_{1,k}(t) - \dot{y}_{r,k}$, $\psi_{1,k}(t) = c_1 z_{1,k} + \hat{\eta}_k \phi(t) x_{1,k}^2 + \frac{1}{\Delta_k}\hat{S}_k x_{1,k}^4 z_{1,k} - \dot{y}_{r,k}$, $\dot{\zeta}_{1,k}(t) = \psi_{1,k}(t) z_{1,k}$; $N(\zeta_{1,k}) = \zeta_{1,k}^2 cos(\zeta_{1,k})$, $\Delta_k = \frac{a}{k^2}$, $\Delta_k = \frac{a}{k^2}$, $\tau_{1,k} = \Gamma_1 \phi(t) x_{1,k}^2 z_{1,k}$, $\nu_{1,k} = \Gamma_2 \frac{1}{\Delta_k} x_{1,k}^4 z_{1,k}^2$.

Step 2. Design

$$\begin{aligned} & u_k(t) = N(\zeta_{2,k}(t))\psi_{2,k}(t), \dot{\zeta}_k(t) = z_{2,k}\psi_{2,k}(t), \\ & \psi_{2,k}(t) = (c_2 - \frac{1}{4}) z_{2,k} - y_{r,k}^{(2)} + \hat{B}_{1,k} z_{2,k} + \hat{\eta}_k^T \phi(t) \frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{1,k}^2 - \frac{\partial \alpha_{1,k}}{\partial x_{1,k}} \hat{b}_{1,k} x_{2,k} \\ & - \frac{\partial \alpha_{1,k}}{\partial \hat{\eta}_k} \tau_{2,k} - \frac{\partial \alpha_{1,k}}{\partial \hat{S}_k} \nu_{2,k} - \hat{\eta}_k^T \dot{\phi}(t) x_{1,k}^2 + \frac{1}{\Delta_k} \hat{S}_k (\frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{1,k}^2)^2 z_{2,k}, N(\zeta_{2,k}) \\ & = \zeta_{2,k}^2 cos(\zeta_{2,k}), \Delta_k = \frac{a}{k^2}, \tau_{2,k} = \tau_{1,k} - \Gamma_1 \phi(t) \frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{1,k}^2 z_{2,k}, \\ & \nu_{2,k} = \nu_{1,k} + \Gamma_2 \frac{1}{\Delta_k} (-\frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{1,k}^2)^2 z_{2,k}^2. \end{aligned} \tag{4.3}$$

Design adaptive laws: $\dot{\hat{\eta}}_k = \tau_{2,k} = \Gamma_1 \phi(t) x_{1,k}^2 z_{1,k} - \Gamma_1 \phi(t) \frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{1,k}^2 z_{2,k}$; $\dot{\hat{S}}_k = \nu_{2,k} = \Gamma_2 \frac{1}{\Delta_k} x_{1,k}^4 z_{1,k}^2 + \Gamma_2 \frac{1}{\Delta_k} (-\frac{\partial \alpha_{1,k}}{\partial x_{1,k}} x_{1,k}^2)^2 z_{2,k}^2$. Related parameters and the initial value are chosen: $a = \frac{100}{3}$, $c_1 = c_2 = 1$, $\Gamma_1 = diag\{0.1\}$, $\Gamma_2 = 10$, $x_{1,0}(0) = 0$, $x_{2,0}(0) = -0.1$, $\hat{\eta}_0(0) = [0.1, 0.1, 0.1, 0.1, 0.1]^T$, $\hat{S}_0(0) = 0.1$, $\zeta_{1,0}(0) = 0$, $\zeta_{2,0}(0) = 0$. Taking $k = 30$, the simulation results are shown in Figs.1-4.

It can be seen from Figs.1-2 that the tracking error can converge to zero. Moreover, Figs.3-6 show that the control signals $\|u_k\|$, $\|\hat{\eta}_k\|$, $\|\hat{S}_k\|$, Nussbaum parameter $\|\zeta_{1,k}\|$, $\|\zeta_{2,k}\|$ and unknown parameters $\|\hat{B}_{1,k}\|$, $\|\hat{b}_{1,k}\|$ are bounded on the interval $[0,1]$. The above results verify the design algorithm in this paper.

5 Conclusions

In this paper, an iterative learning control algorithm is proposed to solve the non-uniform target control problem of nonlinear systems, and deal with the problems of time-varying parameters and multiple unknown control directions at the same time. Finally, through the Lyapunov stability theorem, it is proved that the designed controller can make the tracking error of the closed-loop system converge to zero gradually, can complete the non-uniform target tracking, and this conclusion is proved by an example.

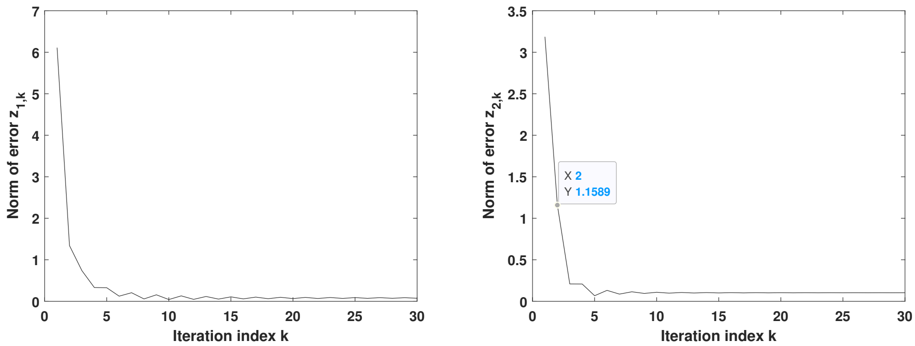

Fig.1-2. The change of $\|z_{1,k}\|$, $\|z_{2,k}\|$ with iteration index.

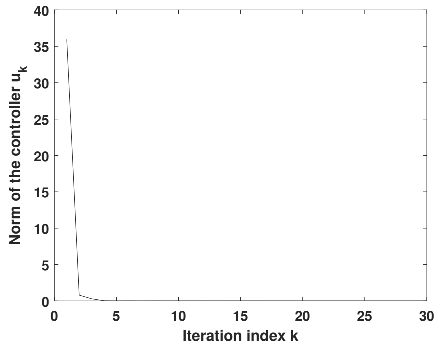

Fig.3. The change of $\|u_k\|$ with iteration index.

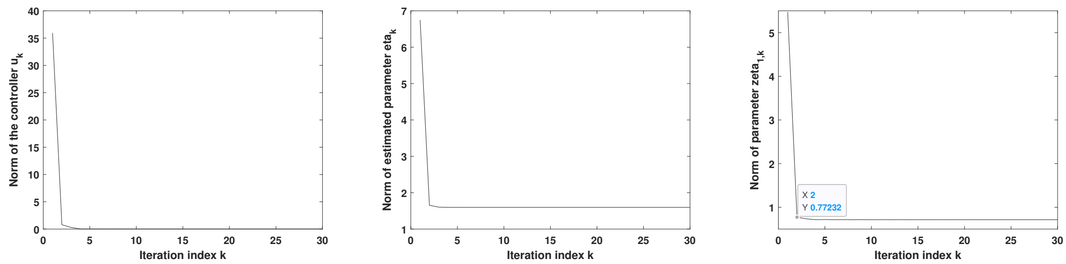

Fig.4. The change of $\|\hat{\eta}_k\|$, $\|\hat{S}_k\|$ and Nussbaum parameter $\|\zeta_k\|$with iteration index.

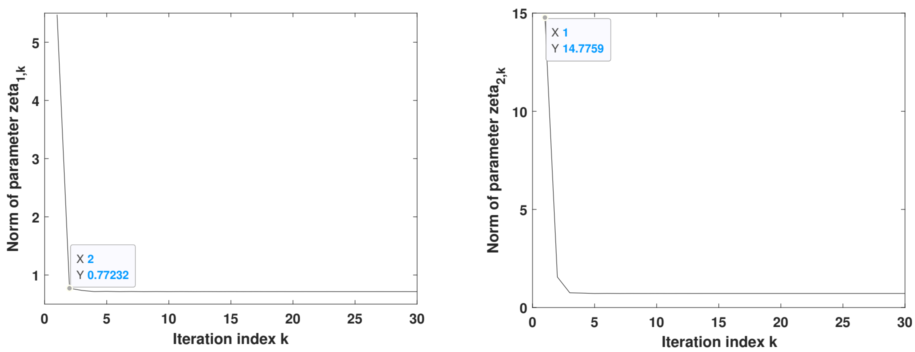

Fig.5. The change of $\|\zeta_{1,k}\|$, $\|\zeta_{2,k}\|$ with iteration index.

6 Declarations

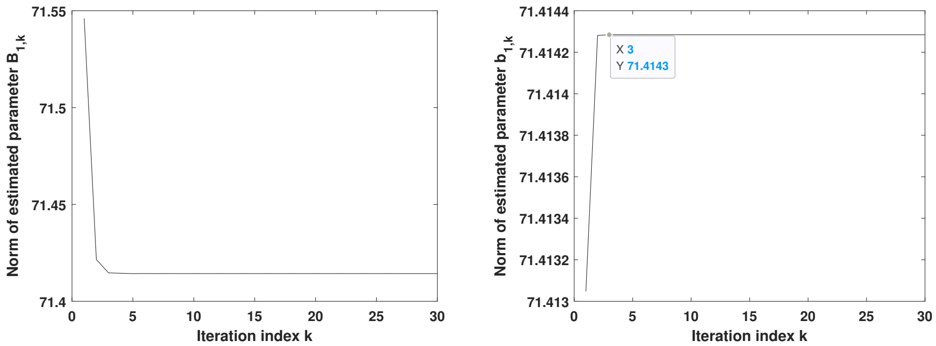

Fig.6. The change of $\|\hat{B}_{1,k}\|$, $\|\hat{b}_{1,k}\|$ with iteration index.

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Competing Interests

The authors declare that they have no competing interests.

Ethical Approval

Not applicable.

Authors' Contributions

All authors contributed equally. All the authors read and approved the final manuscript.

Availability Data and Materials

Not applicable.

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